On *Soft Turing Point with Separation Axioms

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Abstract

In this paper, we use the concept of the *soft turing point and join it with separation axioms in soft topological space and investigate the relationship between them and study the most important properties and results of it.

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1. Introduction and Preliminaries

The concept of soft sets was first introduced by [1] in 1999 as a general mathematical tool for dealing with uncertain objects. [2] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. The notion of soft ideal is initiated for the first time by [3]. [4] studied the soft sets theory as an analytical study and dividing the kinds to four families, to make a comparison between them and identify similarities and differences among them. [5] defined the separation axioms in soft topological space and practically in certain point of the parameters, and to study the most important properties and results of it. [6], first identified the first type of soft set. In addition, [7] and [2] defined of three types of the soft points. In this paper, we chooses one of these families to be the focus of our work is the fourth family (Simply we write the fourth family by \( SS(X) \)) which is introduced by[9]. We define new separation axioms in soft topological space via the concept of the *Soft Turing Point and study the most important properties and results of it.

Throughout this paper, if \((X_A, \bar{A}, A)\) is a soft topological space and \(a \in A, x \in X\), we say that a soft set \(G \in \bar{A}\) is an \(a\)–soft open neighborhood of \(x\) in \((X_A, \bar{A}, A)\) if \(x \in G(a)\) and we denoted by \(G(a,x)\) [8][D.N.Geoigiou,A.C.Magaritis, 2013]. If \(G_a\) is a soft set over the universe \(X\) and \(x \in X\), we say that \((a,x) \in G_a\), whenever \(x \in G(a)\) for all \(a \in A\). That is for any \(x \in X\), \((a,x) \notin G_a\) if \(x \notin G(a)\) for some \(a \in A\).

Simply we write neighborhood by (nhd). The set of all soft open nhd of a point \((a,x)\) is denoted by \(N_{t(a,x)}\)
**Definition 1.1** [1]

Let \( X \) be an initial universe set and \( A \) a set of parameters. A pair \( (F,A) \), where \( F \) is a map from \( A \) to \( P(X) \), is called a soft set over \( X \).
In what follows by \( SS(X,A) \) we denote the family of all soft sets \( (F,A) \) over \( X \).

**Dentition 1.2** [10].

We say that the soft set \( x_a \) in \( SS(X;A) \) is soft point, if for the element \( a \in A \) and \( x \in X, F(a) = \{ x \} \) and \( F(a') = \emptyset \) for every \( a \in A \) - \{a\}.

**Dentition 1.3** [8].

Let \( (F,A), (G,A) \in SS(X,A) \). We say that the pair \( (F,A) \) is a soft subset of \( (G,A) \) if \( F(p) \subseteq G(p) \), for every \( p \in A \). Symbolically, we write \( (F,A) \subseteq (G,A) \). Also, we say that the pairs \( (F,A) \) and \( (G,A) \) are soft equal if \( (F,A) \subseteq (G,A) \) and \( (G,A) \subseteq (F,A) \). Symbolically, we write \( (F,A) = (G,A) \).

**Definition 1.4** [11].

A soft set \( F_A \) over \( \chi \) is said to be the null soft set, denoted by \( \Phi_A \) if \( \forall a \in A, F(a) = \emptyset \).
A soft set \( F_A \) over \( \chi \) is said to be the absolute soft set and denoted by \( \chi_A \), if \( \forall a \in A, F(a) = \chi \).

**Definition 1.5** [12].

Let \( (F,A) \in SS(X,A) \). The soft complement of \( (F,A) \) is the soft set \( (H,A) \in SS(X,A) \), where the map \( H : A \rightarrow P(X) \) defined as follows: \( H(p) = X \setminus F(p) \), for every \( p \in A \).
Symbolically, we write \( (H,A) = (F,A)^c \).

**Definition 1.6** [2].

Let \( X \) be an initial universe set, \( A \) set of parameters, and \( \tilde{\tau} \subseteq SS(X,A) \). We say that the family \( \tilde{\tau} \) defines a soft topology on \( X \) if the following axioms are true:
(1) \( \Phi_A, \chi_A \) belong to \( \tilde{\tau} \)
(2) If \( (G,A), (H,A) \) belong to \( \tilde{\tau} \), then \( (G,A) \cap (H,A) \) belong to \( \tilde{\tau} \)
(3) If \( (G_i,A) \) belong to \( \tilde{\tau} \), for every \( i \in I \), then \( \bigcup\{(G_i,A) : i \in I\} \) belong to \( \tilde{\tau} \)
The triplet \( (X, \tilde{\tau}, A) \) is called a soft topological space or soft space. The members of \( \tilde{\tau} \) are called soft open sets in \( X \). Also, a soft set \( (F,A) \) is called soft closed if the complement \( (F,A)^c \) belongs to \( \tilde{\tau} \). The family of soft closed sets is denoted by \( \tilde{\tau}^c \).

**Definition 1.7** [13].

Let \( (\tilde{X}_A, \tilde{\tau}, A) \) be a soft topological space, and let \( Y \subseteq X \), the relative soft topology for \( \tilde{Y}_A \) is the collection \( \tilde{\tau}_Y \) given by:
\( \tilde{\tau}_Y = \{\tilde{Y}_A \cap F_A : F_A \in \tilde{\tau}\} \).
Note that \( \tilde{Y}_A \) means that \( Y(a) = Y, \forall a \in A \).
The soft topological space \( (\tilde{Y}_A, \tilde{\tau}_Y, A) \) is called soft subspace of \( (\tilde{X}_A, \tilde{\tau}, A) \).
The soft topology \( \tilde{\tau}_Y \) is called induced by \( \tilde{\tau} \).
Definition 1.8 [13]
Let $(\tilde{X}, \tilde{\tau}, A)$ be a soft topological space, and let $G_A$ be a soft set over the universe $\chi$, then:
The soft closure of $G_A$ is a soft closed set defined as:
$\text{Cl}(G_A) = \{S_A, S_A \text{is soft closed and } G_A \subseteq S_A\}$

Proposition 1.9 [7]
Let $(\tilde{X}, \tilde{\tau}, A)$ be a soft topological space, and let $F_A, G_A$ be soft sets over $\chi$, then:
$G_A$ is soft closed iff $\text{Cl}(G_A) = G_A$

Definition 1.10 [5]
Let $\chi$ and $Y$ be two initial universal sets and $A, B$ be sets of parameters, $u: \chi \rightarrow Y$ and $p: A \rightarrow B$, then the mapping:
$f: (\chi, A) \rightarrow (Y, A)$ ( I.e $f: SS(\chi) \rightarrow SS(Y)$) on $A$ and $B$ respectively is denoted by $f_{pu}$ and can be shown as:
$$f_{pu} = \left\{\left(f_{pu}(F_A), p(A) \subseteq B\right)\right\}.$$  
Where:
$$f_{pu}(F_A)(\beta) = \left\{\begin{array}{ll}
\left(u(\bigcup_{\alpha \in p^{-1}(\beta)}(F(\alpha)) \right), & \text{if } p^{-1}(\beta) \neq \emptyset \\
\emptyset, & \text{otherwise}
\end{array}\right.$$  
For $\beta \in B \ni a \in p(A)$ such that $p(a) = \beta$, that is $p^{-1}(\beta) \neq \emptyset$
Since $p^{-1}(\beta) \subseteq A$, hence $p^{-1}(\beta) \cap A \neq p^{-1}(\beta)$, hence we get that
$$f_{pu}(F_A)(\beta) = u\left(\bigcup_{\alpha \in p^{-1}(\beta)} F(\alpha)\right).$$

Constructing:
Since $p$ is a mapping, so $p(A) \neq \emptyset$, $\forall A \neq \emptyset$, that is $\forall \beta \in p(A) \ni a \in A$ such that $p(a) = \beta$ and $p^{-1}(\beta) \neq \emptyset$ since $a \in p^{-1}(p(a))$ so:
$$f_{pu}(F_A)(\beta) = u\left\{\bigcup_{\alpha \in p^{-1}(\beta)} F(\alpha)\right\}$$  
\begin{itemize}
  \item If $p$ is a one to one (1-1), then $p^{-1}(p(A)) = A$, that is $\forall \beta \in p(A) \ni a \in A$ such that $p(a) = \beta$ and $f_{pu}(F_A)(\beta) = u(F(a)).$
  \item If $G_B \in SS(Y)$ then the inverse image of $G_B$ under $f_{pu}$ is denoted by $f_{pu}^{-1}(G_B)$ is a soft set $(F_A) \in SS(\chi)$ such that
\end{itemize}
$P(a) = u^{-1}(G(p(\alpha)))$, for each $a \in A$

Remark 1.11 [5]
For each $a \in A$ and $x \in \chi$, then we can define the soft mapping $f_{pu}$ on a soft point $x_a$, as follows:
\begin{itemize}
  \item $f_{pu}(x_a)_{p(a)} = \{(p(a), \{u(x)\})\}$
  \item Now, for $b \in B$ and $y \in Y$, $f_{pu}^{-1}(y_b)(a) = u^{-1}(y)$, for $b = f(a)$
\end{itemize}
Definition 1.12[14]

For a topological space \((X, T)\), \(x \in X, Y \subseteq X\), we define an ideal \(I_x\) respect to subspace \((Y, T_Y)\), as follows: \(I_x = \{G \subseteq Y : x \in (X-G)\}\).

Definition 1.13: [3]

Let \(I_A\) be a non-null collection of soft sets over a universe \(X\) with the same set of parameters \(A\). Then \(I_A \in SS(X)\) is called a soft ideal on \(X\) with the same set \(A\) if

1- \(F_A \in I_A\) and \(G_A \in I_A\) then \(FA \cup GA \in I_A\)
2- \(F_A \notin I_A\) and \(FA \subseteq GA\) then \(G_A \in I_A\)

Definition 1.14 [8][D. N. Georgiou, A. C. Megaritis , 2013].

Let \((X, \tau, A)\) be a soft topological space, \(a \in A\), and \(x \in X\). We say that a soft set \((F, A) \in \tau\) is an \(a\)-soft open neighborhood of \(x\) in \((X, \tau, A)\) if \(x \in F(a)\).

2- Separation Axioms using *Soft Turing point

Definition 2.1

A point \((a, x)\) in \(A \times X\) is called *Soft Turing point* of a soft ideal \((SI)\) in a soft topological space \((\overline{X_A}, \tau, A)\), if \((G_A) = SI\) \(N_{\tau(a,x)}\) \(A(x)\), for each \(G_A \in N_{\tau(a,x)}\).

Example 2.2

Let \((\overline{X_A}, \tau, A)\) be a soft topological space, \(x \in X\), we define soft ideal \(SI(a, x)\), as follows: \(SI((a, x)) = \{G_A \in N_{\tau(a,x)} : (a, x) \in (G_A)^C\}\). Then point \((a, x)\) is called *Soft Turing point* of \(SI((a, x))\).

Remark 2.3

Let \((\overline{X_A}, \tau, A)\) be a soft topological space and \(a \in A\), For any pair of distinct points \(x_1 \neq x_2\) in \(X\), then following properties are equivalent:

a) \((\overline{X_A} - \{(a, x_2)\}) \in N_{\tau(a,x_2)}\).

b) \((a, x_1)\) is not ** soft Turing point of \(SI(a, x_2)\).

c)

Proof: (a)\(\rightarrow\) (b)

Let \(x_1, x_2 \in X\) such that \(x_1 \neq x_2\). Assume that \((\overline{X_A} - \{(a, x_2)\}) \in N_{\tau(a,x_2)}\), then \((a, x_2)\) is a soft closed set in \(\overline{X_A}\), so that \((a, x_2) = CL\{(a, x_2)\}\). But \(x_1 \neq x_2\), we get that \((a, x_1) \notin CL\{(a, x_2)\}\). Therefore, there exists \(U \in N_{\tau(a,x_1)}\) such that, \((a, x_1) \in U\), \(U \cap \{(a, x_2)\} = \emptyset\). So that \((a, x_1) \in U\), \(U \cap \overline{SI(a, x_2)} = \emptyset\), because if \(U \cap \overline{SI(a, x_2)}\), then \((a, x_2) \in U\), that means \(U \cap \{(a, x_2)\} = \emptyset\), this a contradiction!

Hence \((a, x_1)\) is not * soft Turing point of \(SI(a, x_2)\).

(b)\(\rightarrow\) (a)

Let \(x_1, x_2 \in X\) such that \(x_1 \neq x_2\). Since \((a, x_1)\) is not * soft Turing point of \(SI(a, x_2)\), then there exists \(U \in N_{\tau(a,x_1)}\) such that, \((a, x_1) \in U\), \(U \cap \overline{SI(a, x_2)} = \emptyset\). Thus \((a, x_1) \in U\), \(U \cap \{(a, x_2)\} = \emptyset\) implies \((a, x_1) \notin CL\{(a, x_2)\}\). Hence \((a, x_2) = CL\{(a, x_2)\}\). Thus, \((a, x_2)\) is a soft closed set in \(\overline{X_A}\). Hence \((\overline{X_A} - \{(a, x_2)\}) \in N_{\tau(a,x_2)}\).
Example 2.4
Let $E_X$ be the set of all parameters and let $X$ be the initial universe consisting of:
$X = \{x_1, x_2\}$ and $A \subseteq E_X$ such that $A = \{a_1, a_2\}$.

$\tilde{\tau} = \{\tilde{\phi}_A, \tilde{x}_A, G_{1A}, G_{2A}, G_{3A}, G_{4A}, G_{5A}, G_{6A}, G_{7A}, G_{8A}, G_{9A}, G_{10A}, G_{11A}, G_{12A}, G_{13A}, G_{14A}\}$, where

$G_{1A} = \{(a_1, \{x\}), (a_2, \emptyset)\}$, $G_{2A} = \{(a_1, \emptyset), (a_2, \{x\})\}$

$G_{3A} = \{(a_1, \emptyset), (a_2, \{y\})\}$, $G_{4A} = \{(a_1, \{y\}), (a_2, \emptyset)\}$

$G_{5A} = \{(a_1, \{x\}), (a_2, \{y\})\}$, $G_{6A} = \{(a_1, \{y\}), (a_2, \{x\})\}$

$G_{7A} = \{(a_1, \{x\}), (a_2, \{x, y\})\}$, $G_{8A} = \{(a_1, \{x, y\}), (a_2, \{x\})\}$

$G_{9A} = \{(a_1, \{y\}), (a_2, \{x, y\})\}$, $G_{10A} = \{(a_1, \{x, y\}), (a_2, \emptyset)\}$

$G_{11A} = \{(a_1, \emptyset), (a_2, \{x, y\})\}$, $G_{12A} = \{(a_1, \{x, y\}), (a_2, \emptyset)\}$

$G_{13A} = \{(a_1, \{x\}), (a_2, \{x\})\}$, $G_{14A} = \{(a_1, \{y\}), (a_2, \{y\})\}$

Then, $SI(a_1, x) = \{\tilde{x}_A, G_{2A}, G_{3A}, G_{4A}, G_{5A}, G_{6A}, G_{7A}, G_{8A}, G_{9A}, G_{10A}, G_{11A}, G_{12A}, G_{13A}, G_{14A}\}$.

$SI(a_1, y) = \{\tilde{x}_A, G_{1A}, G_{2A}, G_{3A}, G_{5A}, G_{7A}, G_{11A}, G_{13A}\}$.

$SI(a_2, y) = \{\tilde{x}_A, G_{1A}, G_{2A}, G_{3A}, G_{4A}, G_{6A}, G_{8A}, G_{10A}, G_{13A}\}$.

$SI(a_2, x) = \{\tilde{x}_A, G_{1A}, G_{2A}, G_{3A}, G_{4A}, G_{5A}, G_{8A}, G_{12A}, G_{14A}\}$.

$(a_1, x)$ is a soft turing point of $SI((a_1, y))$, but $(a_1, y)$ is not a soft turing point of $SI(a_1, x)$.

Definition 2.5
Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space and $a \in A$, the space $\tilde{X}_A$ is called a-SI-T$_0$-space if and only if, for any pair of distinct points $x$ and $y$ of $X$, $(a, y)$ is not a soft turing point of $SI((a, x))$ or $(a, x)$ is not a soft turing point of $SI((a, y))$.

Definition 2.6
The soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$ is called SSI-T$_0$-space iff $\forall a \in A$ the soft space $\tilde{X}_A$ is a-SI-T$_0$-space.

Remark 2.7
For a soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$. Every SSI-T$_0$-space is a a-SI-T$_0$-space. [Direct from definition ].

Remark 2.8
The converse, need not be true, as seen in the following example.

Example 2.9
Consider [Example 2.4].
Let $\tilde{\tau} = \{\tilde{\phi}_A, \tilde{x}_A, G_{1A}, G_{2A}\}$ be a soft topology on $\tilde{X}_A$. Then $(\tilde{X}_A, \tilde{\tau}, A)$ is a$_1$-SI-T$_0$-space because, for any pair of distinct points $x$ and $y$ of $X$, there exist a$_1$-soft open set $G_{9A}$ containing $(a_1, y)$ such that $(G_{9A})^c \in SI(a_1, x)$, i.e. $(a_1, y)$ is not a soft turing point of $SI(a_1, x)$, for some $a_1 \in A$. But it not a SSI-T$_0$-space, because, there exist pair of distinct points $x$ and $y$ of $X$ such that $(a_2, y)$ is a soft turing point of $SI(a_2, x)$, $(a_2, x)$ is a soft turing point of $SI(a_1, y)$.
**Theorem 2.10**

A soft subspace of \(a\)-SI-T_0– space is \(a\)-SI-T_0– space , \(\forall \ a \in A \).

**Proof**: Suppose that \(\tilde{Y}_A\) is a soft subspace of the of the a-SI-T_0-space \((\tilde{X}_A, \tilde{\tau}, A)\) and \(a \in A\). Let \(y_1\) and \(y_2\) be two distinct points of \(\tilde{Y}_A\). Again, since \(\tilde{X}_A\) is a-SI-T_0 –space and \(\tilde{Y}_A \subseteq \tilde{X}_A\), then (a,y_1) is not * soft turing point of \( SI (a, y_2) \) or (a,y_2) is not * soft turing point of \( SI (a, y_1) \).

Suppose, (a,y_1) is not * soft turing point of \( SI (a, y_2) \) then there exists \( U \in N_{\tilde{\tau}(a,y_1)} \) such that, (a,y_1) \( \in U \), \( U^c \in SI (a, y_2) \). Then \( U = U \cap \tilde{Y}_A \) is \( \tilde{\tau}_Y\)- soft open contains (a,y_1) but not (a,y_2). So that (a,y_1) \( \in U \) and \( (U)^c \in SI (a, y_2) \), hence \( \tilde{Y}_A \) is a-SI-T_0-space.

**Theorem 2.11**

Let \((\tilde{X}_A, \tilde{\tau}, A)\) and \((\tilde{Y}_D, \tilde{\sigma}, D)\) be two soft topological spaces and let \(\tilde{X}_A\) be a a-SI-T_0-space , for some \(a \in A\), if the map \(f_{dv}: (\tilde{X}_A, \tilde{\tau}, A) \rightarrow (\tilde{Y}_D, \tilde{\sigma}, D)\) is a soft open and , \(d, v\) are onto maps , then \(\tilde{Y}_D\) is d(a)-SI-T_0-space.

**Proof**: Let \(d \in D\) and \(y_1 \neq y_2\) in \(Y\), then there exist \(a \in A\) and \(x_1 \neq x_2\) in \(X\) such that \(\nu(x_1) = y_1\) and \(\nu(x_2) = y_2\), \(d(a) = b\), because \(d\) and \(v\) are onto maps . Now by assumption , then (a,x_1)is not * soft turing point of \( SI (a, x_2) \) or (a,x_2)is not * soft turing point of \( SI (a, x_1) \), then , there exist \(G_A \in N_{\tilde{\tau}(a,x_1)}, U \in N_{\tilde{\tau}(a,x_2)}\) such that \((a,x_1) \in G_A, (G_A) \in SI (a, x_2)\) or \((a,x_2) \in U_A, (U_A) \in SI (a, x_1)\). Now: \((f_{dv}(a,x_1))) \in f_{dv}(G_A), (f_{dv}(G_A)) \in SI (f_{dv}(a,x_2))\) or \((f_{dv}(a,x_2)) \in f_{dv}(U_A), (f_{dv}(U_A)) \in SI (f_{dv}(a,x_1))\), but \(f_{dv}\) is soft open, so \(f_{dv}(G_A), f_{dv}(U_A)\) are be a soft open sets in \(\tilde{Y}_B\), and \((b,y_1) = (d(a), \nu(x_1)) = f_{dv}(a,x_1)\) and \((b,y_2) = (d(a), \nu(x_2)) = f_{dv}(a,x_2)\). i.e \((b,y_1)\) is not * soft turing point of \( SI (b, y_2)\) or \((b,y_2)\) is not * soft turing point of \( SI (b, y_1)\). Therefore , \(\tilde{Y}_B\) is b-SI-T_0-space.

**Theorem 2.12**

Let \((\tilde{X}_A, \tilde{\tau}, A)\) be a soft topological space and \(a \in A\),then the following properties are equivalent:

\(d)\) \((\tilde{X}_A, \tilde{\tau}, A)\) is a-SI-T_0-space.

\(e)\) For any pair of distinct points \(x\) and \(y\) of \(\tilde{X}_A\) then \(Cl\{ (a, x_1) \} \neq Cl\{ (a, x_2) \}\).

**Proof :** \((a)\rightarrow (b)\)

Suppose that \((\tilde{X}_A, \tilde{\tau}, A)\) is a-SI-T_0-space for some \(a \in A\) and \(x_1 \neq x_2\) in \(X\), then \((a,x_1)\)is not * soft turing point of \( SI (a, x_2) \) or \((a,x_2)\)is not * soft turing point of \( SI (a, x_1)\),so there exist \(G_A \in N_{\tilde{\tau}(a,x_1)}, U \in N_{\tilde{\tau}(a,x_2)}\) such that \((a,x_1) \in G_A, (G_A) \in SI (a, x_2)\) or \((a,x_2) \in U_A, (U_A) \in SI (a, x_1)\). Then , by Remark 2.3, then \(Cl\{ (a, x_1) \} = (a, x_1)\) or \(Cl\{ (a, x_2) \} = (a, x_2)\). That means \((a, x_1) \in Cl\{ (a, x_1) \}\) and \((a, x_2) \in Cl\{ (a, x_2) \}\). Thus, \((a, x_1) \in Cl\{ (a, x_1) \}\) but \((a, x_1) \notin Cl\{ (a, x_2) \}\). Hence, \(Cl\{ (a, x_1) \} \neq Cl\{ (a, x_2) \}\).

\((b)\rightarrow (a)\): Let \(a \in A\) and \(x_1 \neq x_2\) in \(X\), with \(Cl\{ (a, x_1) \} \neq Cl\{ (a, x_2) \}\), then there exist \((a,z) \in Cl\{ (a, x_1) \}\), but \((a,z) \notin Cl\{ (a, x_2) \}\), then \((a, x_1) \notin Cl\{ (a, x_2) \}\) because, if \((a, x_1) \in Cl\{ (a, x_2) \}\), then \(Cl\{ (a, x_1) \} \subseteq Cl(Cl\{ (a, x_2) \}) = Cl\{ (a, x_2) \}\) but \((a,z) \in Cl\{ (a, x_1) \} \subseteq Cl\{ (a, x_2) \}\) which is a contradiction!, thus \((a, x_1) \notin Cl\{ (a, x_2) \}\) ,which implies that \((a, x_1) \in (\tilde{X}_A - Cl\{ (a, x_2) \}) \subseteq N_{\tilde{\tau}(a,x_1)}\) such that \((a, x_1) \in (\tilde{X}_A - Cl\{ (a, x_2) \})\), so \(Cl\{ (a, x_2) \} \notin SI (a, x_2)\). Hence \((\tilde{X}_A, \tilde{\tau}, A)\) is a-SI-T_0-space.
Theorem 2.13

Let \((\tilde{Y}_B, \tilde{\sigma}, B)\) be \(b\)-SI-T_0-space for \(b \in B\) and let \((\tilde{X}_A, \tilde{\tau}, A)\) be any soft topological space such that the mapping \(u: X \rightarrow Y\) be a one to one and \(p: A \rightarrow B\) be an onto map, then there exist \(a \in A\) with \(p(a) = b\) and \(\tilde{X}_A\) is a-SI-T_0-space, if \(f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \rightarrow (\tilde{Y}_B, \tilde{\sigma}, B)\) is a \(-\)soft continuous map.

Definition 2.14

Let \((\tilde{X}_A, \tilde{\tau}, A)\) be a soft topological space and \(a \in A\), the space \(\tilde{X}_A\) is called a-SI-T_1-space if and only if, for any pair of distinct points \(x\) and \(y\) of \(X\), \((a, y)\) is not \(*\) soft turing point of \(SI((a, x))\) and \((a, x)\) is not \(*\) soft turing point of \(SI((a, y))\).

Definition 2.15

The soft topological space \((\tilde{X}_A, \tilde{\tau}, A)\) is called SSI-T_1-space iff \(\forall a \in A\) the soft space \(\tilde{X}_A\) is a-SI-T_1-space.

Remark 2.16

For a soft topological space \((\tilde{X}_A, \tilde{\tau}, A)\). Every SSI-T_1-space is a a-SI-T_1-space. [Direct from definition].

Remark 2.17

The converse, need not be true, as seen in the following example.

Example 2.18

Consider [Example 2.4] Let \(\tilde{\tau} = \{\tilde{\sigma}_A, \tilde{X}_A, G_{13} A\}\) be a soft topology on \(\tilde{X}_A\). Then \((\tilde{X}_A, \tilde{\tau}, A)\) is a_1-SI-T_1-space, but not SSI-T_1-space

Remark 2.19

Every a-SI-T_1-space is a-SI-T_0-space, but the converse is not true.

Proof Direct from [Def]. The converse, need not be true, as seen in the following example.

Example 2.20

Consider [Example 2.4 ] Let \(\tilde{\tau} = \{\tilde{\sigma}_A, \tilde{X}_A, G_{13} A\}\) be a soft topology on \(\tilde{X}_A\). Then \((\tilde{X}_A, \tilde{\tau}, A)\) is a_1-SI-T_0-space, but not a_1-SI-T_1-space

Theorem 2.21

A soft subspace of a-SI-T_1- space is a-SI-T_1- space , \(\forall a \in A\).

Proof: Similar to Theorem2.10

Theorem 2.22

Let \((\tilde{X}_A, \tilde{\tau}, A)\) and \((\tilde{Y}_B, \tilde{\sigma}, B)\) be two soft topological spaces and let \(\tilde{X}_A\) be a-SI-T_1-space , for some \(a \in A\), if the map \(f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \rightarrow (\tilde{Y}_B, \tilde{\sigma}, B)\) is a soft open and \(u, p\) are onto maps, then \(\tilde{Y}_B\) is p(a)-SI-T_1-space.

Proof: Similar to Theorem2.11.
Theorem 2.23
Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space and $a \in A$, $x \in X$, then the following properties are equivalent:

f) $(\tilde{X}_A, \tilde{\tau}, A)$ is a $\text{SI-T}_0$-space.

g) $(\tilde{X}_A - \{(a, x)\}) \notin N_{t(a,x)}$

Proof: Follows from Remark 2.3.

Theorem 2.24
Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space and $a \in A$, then the following properties are equivalent:

a) $(X,T)$ is a $\text{SI-T}_1$-space.

b) For any $x \neq y$ in $\tilde{X}_A$ and $a \in A$, $(\tilde{X}_A - \{(a, x)\}) \notin N_{t(a,x)}$ and $(\tilde{X}_A - \{(a, y)\}) \notin N_{t(a,y)}$

Proof: Follows from definition, Remark 2.3.

Definition 2.25
Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space and $a \in A$, the space $\tilde{X}_A$ is called a $\text{SI-T}_1$-space if and only if, for any pair of distinct points $x$ and $y$ of $X$, $(a, y)$ is not * soft turing point of $\text{SI}((a, x))$ and $(a, x)$ is not * soft turing point of $\text{SI}((a, y))$, $\text{SI}(a, y) \cap \text{SI}(a, x) = \emptyset$.

Remark 2.27
For a soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$. Every a-$\text{SI-T}_2$-space is a a-$\text{SI-T}_1$-space. But The converse, need not be true.

Example 2.29
Consider [Example 2.4]
Let $\tilde{\tau} = \{\varphi_A, \tilde{X}_A, G_7_A, G_4_A\}$ be a soft topology on $\tilde{X}_A$. Then $(\tilde{X}_A, \tilde{\tau}, A)$ is $a_1$-$\text{SI-T}_1$-space, but not $a_1$-$\text{SI-T}_2$-space

Theorem 2.30
Every soft subspace of a-$\text{SI-T}_2$-space is a-$\text{SI-T}_2$-space $\forall a \in A$.

Proof: Similar to Theorem 2.10.

Theorem 2.31
Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces and let $\tilde{X}_A$ be a-$\text{SI-T}_2$-space, for some $a \in A$, if the map $f_{pu}(\tilde{X}_A, \tilde{\tau}, A) \rightarrow (\tilde{Y}_B, \tilde{\sigma}, B)$ is a soft open and $u, p$ are onto maps, then $\tilde{Y}_B$ is p(a)-$\text{SI-T}_2$-space.

Proof: Similar to Theorem 2.11.

References


نقطة التحول الطريفة (٩) مع بديهيات الفصل

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الخلاصة:
في هذا البحث استخدمنا مفهوم نقطة التحول الطريفة (٩) وربطها مع بديهيات الفصل في الفضاء التوولوجي الطري. وبحث العلاقة بينهما ودراسة أهم الخصائص والنتائج لها.

الكلمات المفتاحية: بديهيات الفصل، نقطة تحول طريفة (٩).