On Compactly Fuzzy Generalized b-k-closed Sets

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Abstract

In this paper ,we have dealt with the concepts of fuzzy generalized b- irresolute (in short fgb-irresolute),fuzzy strongly generalized b-compact (in short fstg-b-compact) and fuzzy generalized b-continuous (in short fgb-continuous) functions , and find that every fgb-irresolute function is fgb-continuous and the converse is not true in general .We have also remember the concept of fuzzy T_2-space (fT_2-space) and we have proved that every fuzzy generalized b-compact (in short fgb-compact) subset of an fT_2-space is fuzzy generalized b-closed (in short fgb-closed) , and if X isn't fT_2- space , then need not that every fgb-compact set is fgb-closed . Finally we defined compactly fuzzy generalized b-closed (in short compactly fgb-closed) set and proved that every fgb-closed subset of a space X is compactly fgb-closed .

Keywords: fuzzy b-open ,compactly fuzzy b-closed set, compactly fuzzy b-k-closed set .

الخلاصة

في بحثنا هذا تعاملنا مع تعميم الدوال الضبابية المترددة –b, الضبابية المرصوصة بقوة –b والضبابية المستمرة –b , ووجدنا أن تعميم كل ضبابية مترددة –b تكون ضبابية مستمرة –b وان العكس غير صحيح دائماً . أيضاً تناولنا مفهوم الفضاء الضبابي T_2 وبرهنا انه في الفضاء T_2 تعميم كل مجموعة ضبابية مرصوصة –b تكون ضبابية مغلقة –b , واستنتجنا انه إذا لم يكن الفضاء من نوع T_2 فليس شرط أن يكون تعميم كل مجموعة ضبابية مرصوصة –b ضبابية مغلقة –b , وعرفنا ايضاً تعميم المجموعة الضبابية المغلقة –b–K بشكل مرصوص وبرهنا ان تعميم كل ضبابية مغلقة –b–K بشكل مرصوص في الفضاء التبولوجي الضبابي تكون تعميم ضبابية مرصوصة مغلقة –b .

الكلمات المفتاحية: المجموعات الضبابية المفتوحة –b , المجموعة الضبابية المغلقة –b بشكل مرصوص , المجموعة الضبابية المغلقة –b–K بشكل مرصوص .

Introduction

Zadeh in **[Zadeh, 1965]** introduced the fundamental concept of fuzzy sets. The study of fuzzy topology was introduced by Chang in **[Chang, 1968]**.The theory of fuzzy topological spaces was subsequently developed by several authors. In 1970Levine **[Levine,1970]** first considered the concept of generalized closed (briefly, g-closed)sets were defined and investigated.In 1969 **[Andrijevic, 1996]** introduced a class of generalized open sets in a topological space called b-open sets. **[Omari and Noorani, 2009]** introduced and studied the concept of fuzzy generalized b-closed sets (briefly fgb-closed) in topological spaces. In this paper, we introduced and studied the concepts of compactly fgb-closed set and compactly fgb-k-closed set in fuzzy topological spaces.Throughout this paper X and Y mean fuzzy topological spaces.This paper includes three sections . In the first section, we recall the concepts of fuzzy b-open,fuzzy b-closed set and b-quasi neighborhood, in the second section we have dealt with the concepts of fuzzy gb-open ,fuzzy gb-closed set, fuzzy net and some their propositions, in section three we dealt with fuzzy b-compact space, fuzzy gb-compact space, fuzzy gb-irresolute function and some theorems related to them.

1. Preliminaries

In this section, we review some basic definitions, propositions and theorems about some concepts which are needed in next chapter.

Definition 1.1 [Dang, *et al.*,1994]

A fuzzy point x_α in X is a fuzzy set defined as follows :

$x_\alpha(y) = \begin{cases} \alpha & if \quad y = x \\ 0 & if \quad y \neq x \end{cases}$ where $0 < \alpha \leq 1$; α is called a value of x_α and x is called its support.

The set of all fuzzy points in X will be denoted by FP(X).

Remark 1.2 [Dang *et al.*, 1994]

Two fuzzy points x_α and y_β in X are said to be distinct if and only if their supports are distinct (i.e $x \neq y$).

Definition 1.3 [Rashid and Ali, 2008]

A fuzzy point x_α is said to belong to a fuzzy set A in X (denoted by : $x_\alpha \in A$) if and only if $\alpha \leq A(x)$.

Proposition 1.4 [Dang, *et al.*,1994]

Let A and B be fuzzy sets in X . Then A subset of B (denoted by : $A \leq B$) if and only if $x_\alpha \in A$, then $x_\alpha \in B$.

Definition 1.5 [Benchalli and Jenifer, 2011]

A fuzzy point x_α is called quasi-coincident with a fuzzy set A, denoted by $x_\alpha q A$ if and only if there exists $x \in X$ such that $\alpha + A(x) > 1$.

Definition 1.6 [Nouh, 2005]

A fuzzy set A in X is called quasi-coincident with a fuzzy set B, denoted by $A\,q\,B$ if and only if $A(x) + B(x) > 1$, for some $x \in X$ .If A is not quasi-coincident with B, then $A(x) + B(x) \leq 1$ for every $x \in X$, and denoted by $A\,\overline{q}\,B$.

Proposition 1.7 [Zahran,1989]

Let A and B are fuzzy sets in X , then :

i) $x_\alpha \in A$ if and only if $x_\alpha\,\overline{q}\,A^c$.

ii) $A \leq B$ if and only if $A\,\overline{q}\,B^c$.

iii) $A \leq B$ if and only if $x_\alpha\,q\,B$ for each $x_\alpha q\,A$.

iiii) $A\,q\,B$ if and only if $A \leq B^c$.

Proposition 1.8 [Mohammed, 2011]

Let A be a fuzzy set in X , a fuzzy point $x_\alpha \in cl(A)$ if and only if for every fuzzy open set B in X , if $x_\alpha\,q\,B$, then $A\,q\,B$.

Definition 1.9 [Benchalli and Jenifer, 2010]
A fuzzy set A in X is called:
i) fuzzy b-open (in short fb-open) set if and only if $A \leq (\operatorname{int}(cl(A)) \vee cl(\operatorname{int}(A)))$.
ii) fuzzy b-closed (in short fb-closed) set if and only if $(\operatorname{int}(cl(A)) \vee cl(\operatorname{int}(A))) \leq A$.

Definition 1.10 [Benchalli and Jenifer, 2010]
A fuzzy set A in X is called :
i) b-neighborhood of a fuzzy point x_α in X if there exists fb-open set B in X such that $x_\alpha \in B \leq A$.
ii) b-quasi neighborhood of a fuzzy point x_α in X if there exists fb-open set B such that $x_\alpha q B \leq A$.
The family $N_{x\alpha}^{bq}$ consisting of all b-quasi neighborhoods of x_α is called the system of b-quasi neighborhoods of x_α .

Theorem 1.11 [Benchalli and Jenifer ,2010]
Let A be a fuzzy set in X , then a fuzzy point $x_\alpha \in bcl(A)$ if and only if every b-quasi neighborhoods of x_α is quasi-coincident with A, where $bcl(A) = \wedge\{B : A \leq B, B \text{ is fb-closed set}\}$.

2. Fuzzy Generalized b-closed (b-open) set .
This section will contain the concept of fuzzy generalized b-closed (b-open) set with some of its properties that are necessary to the work.

Definition 2.1 [Benchalli, 2011]
A fuzzy set A in X is called fuzzy generalized b-closed (in short, fgb-closed) set if $bcl(A) \leq B$ where $A \leq B$ and B is fuzzy open set .

Remarks 2.2 [**Benchalli and Jenifer,2010**]
i) A fuzzy set A in X is called fgb-open if its complement is fgb-closed.
ii) Every f-closed (f-open) set is fb-closed (fb-open) set, and every fb-closed (fb-open) set is fgb-closed (fgb-open), but the converse is not true.

Definition 2.3
Let A be a fuzzy set in X , the intersection of all fgb-closed sets containing A is called a gb-closure of A and denoted by gbcl(A) .
i.e $gb\,cl(A) = \wedge\{B : A \leq B, B \text{ is a fgb-closed set}\}$.

Definition 2.4
A fuzzy set A in X is called gb-quasi neighborhood of a fuzzy point x_α in X if there exists fgb-open set B such that $x_\alpha q B \leq A$.

Definition 2.5

Let x_α be a fuzzy point in X, then the family $N_{x\alpha}^{gbq}$ consisting of all gb-quasi neighborhoods of x_α is called the system of gb-quasi neighborhoods of x_α .

Definition 2.6 [Nouh,2005]

A mapping $S: D \to FP(X)$ is called a fuzzy net in X and is denoted by $\{S(n): n \in D\}$ where D is a directed set , if $S(n) = x_{\alpha_n}^n$ for each $n \in D$, $x \in X$,and $\alpha_n \in (0,1]$,then the fuzzy net S is denoted as $\{x_{\alpha_n}^n : n \in D\}$ or simply $\{x_{\alpha_n}^n\}$.

Definition 2.7 [Nouh,2005]

A fuzzy net $\varsigma = \{y_{\alpha_m}^m : m \in E\}$ in X is said to be fuzzy subnet of a fuzzy net $S = \{x_{\alpha_n}^n : n \in D\}$ if and only if there is a function $f : E \to D$ such that

1) $\varsigma = S \circ f$,that is $y_{\alpha_i}^i = x_{\alpha_{f(i)}}^{f(i)}$ for each $i \in E$,where E is a directed set .

2) for each $n \in D$,there exist some $m \in E$ such that $f(m) \geq n$.

We shall denoted a fuzzy subnet of a fuzzy net $\{x_{\alpha_n}^n : n \in D\}$ by $\{x_{\alpha_{f(m)}}^{f(m)} : m \in E\}$.

Definition 2.8 [Nouh,2005]

Let (X,T) be a fuzzy topological space and let $S = \{x_{\alpha_n}^n : n \in D\}$ be a fuzzy net in X and $A \in I^X$ .Then S is said to be :

1) Eventually with A if and only if $\exists m \in D$ such that $x_{\alpha_n}^n qA$, $n \geq m$.

2) Frequently with A if and only if $\forall n \in D$, $\exists m \in D$, $m \geq n$, and $x_{\alpha_m}^m qA$.

Definitions 2.9

Let $S = \{x_{\alpha_n}^n : n \in D\}$ be a fuzzy net in X and $x_\alpha \in FP(X)$ .Then S is said to be :

i) gb-convergent to x_α and denoted by $S \overset{gb}{\to} x_\alpha$,if $\forall A \in N_{x_\alpha}^{gbq}$, $\exists m \in D$ such that $x_{\alpha_n}^n qA$, $\forall\, n \geq m$, x_α is called gb-limit point of S .

ii) has a gb-cluster point x_α and denoted by $S \overset{\mathrm{gb}}{\alpha} x_\alpha$, if $\forall A \in N_{x_\alpha}^{gbq}$ and $\forall n \in D, \exists m \in D, m \geq n$ such that $x_{\alpha_m}^m qA$.

Remark 2.10

i) if $x_{\alpha_n}^n$ is a fuzzy net in X $x_{\alpha_n}^n \overset{b}{\to} x_\alpha, x_\alpha \in FP(X)$ such that $x_{\alpha_n}^n \overset{gb}{\to} x_\alpha$ then $x_{\alpha_n}^n \overset{gb}{\alpha} x_\alpha$.

ii) if $x_{\alpha_n}^n$ is a fuzzy net in X , $x_\alpha \in FP(X)$ such that $x_{\alpha_n}^n \to x_\alpha$,then $x_{\alpha n}^n \overset{b}{\to} x_\alpha$ and $x_{\alpha_n}^n \overset{gb}{\to} x_\alpha$.

iii) if $x_{\alpha_n}^n$ is a fuzzy net in X , $x_\alpha \in FP(X)$ such that $x_{\alpha_n}^n \;\alpha\; x_\alpha$,then $x_{\alpha_n}^n \;\overset{b}{\alpha}\; x_\alpha$ and $x_{\alpha_n}^n \;\overset{gb}{\alpha}\; x_\alpha$.

Proposition 2.11

Let A be a fuzzy set in X , and let $x_\alpha \in FP(X)$. If there exists a fuzzy net $x^n_{\alpha_n}$ in A such that $x^n_{\alpha_n} \xrightarrow{gb} x_\alpha$, then $x_\alpha \in gbcl(A)$.

Proof:-

Let $x^n_{\alpha_n}$ be a fuzzy net in A such that $x^n_{\alpha_n} \xrightarrow{gb} x_\alpha$,then for every $B \in N^{gbq}_{x_\alpha}$,there exists $m \in D$ such that $x^n_{\alpha_n} qB$ for all $n \geq m$,see definition 2.8 (i).

Since $x^n_{\alpha_n} \in A$, then $x^n_{\alpha_n} \tilde{q} A^c$, see proposition 1.7(i).

Then $A \leq B$ see proposition 1.7 (iii), i.e $B \leq A^c$

Then AqB , see proposition 1.7 (iiii).

Therefore $x_\alpha \in bcl(A)$ see theorem 1.11

Then $x_\alpha \in gbcl(A)$ see remark 2.2 (i)□.

3.The main Results

In this section, we defined and study new forms of fuzzy generalized b-compact set.

Definition 3.1

A family $F = \{C_\alpha : \alpha \in \Omega\}$ of fuzzy sets in X is called a cover of a fuzzy set A if and only if $A \leq \underset{\alpha \in \Omega}{\vee} C_\alpha$, and it is called a fgb-open cover if each member C_α is a fgb-open set .A sub cover of A is a sub family of F which is also a cover of A .

Definition 3.2

Let B be a fuzzy set in X .Then B is said to be a fgb-compact set if for every fgb-open cover $\{C_\alpha : \alpha \in \Omega\}$ of B has a finite sub cover .Let $B = X$, then X is called a fgb-compact space if for every $\alpha \in \Omega$ and $\underset{\alpha \in \Omega}{\vee} C_\alpha = 1_X$, then there are finite many indices $\alpha_1, \alpha_2, ..., \alpha_n \in \Omega$ such that $\overset{n}{\underset{i=1}{\vee}} B_{\alpha_i} = 1_X$.

Proposition 3.3

Every fgb-compact set in X is fb-compact .

Proof:-

Let A be a fgb-compact set, and let $\{C_\alpha : \alpha \in \Omega\}$ be a fb-open cover of A .

Then $A \leq \underset{\alpha \in \Omega}{\vee} C_\alpha$.

Since every fb-open set is a fgb-open set, see remarks 2.2 (ii) .

Then $\{C_\alpha : \alpha \in \Omega\}$ is a fgb-open cover of A .

Since A is fgb-compact set, then there are finite many indices $\alpha_1, \alpha_2, ..., \alpha_n \in \Omega$ such that $A \leq \overset{n}{\underset{i=1}{\vee}} C_{\alpha_i}$.

Then A is fb-compact set □.

Definition 3.4 [A.S.Mashour,E.E.Kerre and M.H.Ghanim,1984]

A fuzzy topological space X is said to be f-Hausdorff or fT_2-space, if for every pair of fuzzy points x_χ, y_α with different support, there exist two fb-open sets U and V such that $x_\chi \in U \leq (y_\alpha)^c$, $y_\alpha \in V \leq (x_\chi)^c$ and $U\tilde{q}V$.

Theorem 3.5 [Ali,2014]

Every fb-compact set of a fT_2- space is fb-closed set.

Theorem 3.6

Every fgb-compact set of a fT_2- space is fgb-closed set.

Proof:-

Let A be a fgb-compact set in a fT_2-space X .
Then A is a fb-compact set. see proposition **(**3.3**)** .
Then by theorem (3.5), we have A is a fb-closed set .
Since every fb-closed set is fgb-closed, see Remarks 2.2 (ii) .
Then A is fgb-closed set □.

If X dosn't fT_2-space, then need not that every fgb-compact set is fgb-closed set as the following example :-

Example 3.7

Let $X = \{a,b\}$ and let $T = \{0_X, 1_X, A\}$ be a fuzzy topology on X where $A : X \to [0,1]$ defined by $A(a) = 0.2, \quad A(b) = 0.7$
Then A is fgb-compact set, but not fgb-closed set .

Proposition 3.8 [Ali,2014]

A fuzzy topological space X is fb-compact if and only if every fuzzy net in X has a b-convergent fuzzy subnet.

Theorem 3.9

A fuzzy topological space X is fgb-compact if and only if every fuzzy net in X has gb-convergent fuzzy subnet .

Proof:-

By proposition **(**3.3**)**, proposition **(**3.8**)** and remark **(**2.10 (ii) **)** □.

Theorem 3.10

In any fuzzy space, the intersection of a fgb-compact set with a fgb-closed set is fgb-compact.

Proof:-

Let A be a fgb-compact set and B be a fgb-closed set in X .
Let $x_{\alpha_n}^n$ be a fuzzy net in $A \wedge B$, then $x_{\alpha_n}^n$ be a fuzzy net in A and $x_{\alpha_n}^n$ be a fuzzy net in B .
Since $x_{\alpha_n}^n$ is a fuzzy net in A and A is a fgb-compact.

Then by theorem (3.9) we have $x_\alpha \in A$ and $x^n_{\alpha_n} \xrightarrow{gb} x_\alpha$.

Since $x^n_{\alpha_n}$ is a fuzzy net in B such that $x^n_{\alpha_n} \xrightarrow{gb} x_\alpha$.

Then $x_\alpha \in gbcl(\mathrm{B})$, see proposition (2.11)

Since B is fgb-closed, then $gbcl(B) = B$, therefore $x_\alpha \in B$.

Then $x_\alpha \in A \wedge B$ and $x^n_{\alpha_n} \xrightarrow{gb} x_\alpha$.

Then $A \wedge B$ is fgb-compact set □.

Definition 3.11

A function $f : X \to Y$ is called fuzzy generalized b-continuous (in short fgb-continuous), if $f^{-1}(A)$ is fgb-closed set in X for every f-closed set A in X .

Theorem 3.12

A function $f : X \to Y$ is fgb-continuous if and only if the inverse image of each f-open set in Y is fgb-open set in X .

Definition 3.13

A function $f : X \to Y$ is said to be fuzzy generalized b-irresolute (in short fgb-irresolute) if the inverse image of each fgb-closed set in Y is fgb-closed set in X .

lemma 3.14

Every fgb-irresolute function is fgb-continuous

Proof:-

Let $f : X \to Y$ be fgb-irresolute and let A be f-closed set in Y.

Since every f-closed set is fgb-closed set, see remark 2.2 (ii), and since $f : X \to Y$ is fgb-irresolute.

Then $f^{-1}(A)$ is fgb-closed set in X .

Then $f : X \to Y$ is fgb-continuous □.

The converse is not true as the following example.

Example 3.15

Let $X = \{a,b\}$, $Y = \{x, y\}$,and let $T = \{0_x, 1_x, A\}$, $T' = \{0_y, 1_y, B\}$, where $A : X \to [0,1]$ defined by $A(a) = 0.6,\ \ A(b) = 0.3$,and $B : Y \to [0,1]$ defined by $B(x) = 0.5,\ \ B(y) = 0.2$.

$bO(X) = \{0_x, 1_x, ((a,\alpha),(b,\beta))\}$ where $\alpha > 0.4$ or $\beta > 0.7$.

$bO(Y) = \{0_y, 1_y, ((x,\alpha^*),(y,\beta^*))\}$ where $\alpha^* > 0.5$ or $\beta^* > 0.8$.

Then the function $f : X \to Y$ defined by $f(a) = x,\ \ f(b) = y$ is fgb-continuous but not fgb-irresolute.

Definition 3.16

A function $f : X \to Y$ is called a strongly fuzzy generalized b-compact (in short sfgb-compact) function if and only if $f^{-1}(A)$ is fgb-compact set in X for every fgb-compact set A in Y.

Proposition 3.17
Let $f: X \to Y$ be a fgb-irresolute function, if A is fgb-compact in X, then $f(A)$ is fgb-compact set in Y .
Proof:-
Let $\{C_\alpha : \alpha \in \Omega\}$ be a fgb-open cover of $f(A)$.
Then $f(A) \le \vee_{\alpha\in\Omega} C_\alpha$.
Since f is fgb-irresolute function, then $f^{-1}(A)$ is fgb-open set in X .
Then the collection $\{f^{-1}(C_\alpha) : \alpha \in \Omega\}$ is a fgb-open cover of A.
Then $A \le f^{-1}(f(A)) \le f^{-1}(\vee_{\alpha\in\Omega} C_\alpha) = \vee_{\alpha\in\Omega} f^{-1}(C_\alpha)$
Since A is fgb-compact set in X , then there are finite many indices $\alpha_1, \alpha_2, ..., \alpha_n \in \Omega$ such that $A \le \vee_{i=1}^{n} f^{-1}(C_{\alpha i})$.
Then $f(A) \le f(\bigvee_{i=1}^{n} f^{-1}C_{\alpha i}) = \bigvee_{i=1}^{n} f(f^{-1}(C_{\alpha i})) \le \bigvee_{i=1}^{n} C_{\alpha i}$.
Then $f(A)$ is fgb-compact set □.

Definition 3.18
A fuzzy subset W of X is called compactly fuzzy generalized b-closed (in short cfgb-closed) set if $W \wedge K$ is fgb-compact set for every fgb- compact set K in X .

Example 3.19
Every fuzzy subset A of indiscrete fuzzy space is cfgb-closed set .

Proposition 3.20
Every fgb-closed subset of X is cfgb- closed set .
Proof :-
Let A be a fgb- closed subset of X , and let K be a fgb- compact subset in X .
Then by theorem **(**3.10**)** , we have $A \wedge K$ is fgb- compact set .
Then A is cfgb- closed set □.

The converse of above proposition is not true in general as the following example .
Example 3.21
Let $X = \{a, b\}$ and let T be the indiscrete fuzzy space on X .
Then $A: I \to X$ which is defined by $A(a) = 0.1, \quad A(b) = 0.2$ is cfgb-closed set, but it is not fgb-closed set .

Proposition 3.22
Let $f: X \to Y$ be a fgb-irresolute , sfgb-compact, bijective function, then A is cfgb-closed subset in X if and only if $f(A)$ is cfgb-closed set in Y .
Proof :-
$\Rightarrow$ Let A be cfgb-closed set in X , and let K be a fgb-compact set in Y .
Since f is sfgb-compact function, then $f^{-1}(k)$ is fgb- compact set in X .
Since A is cfgb-closed set in X

Then $A \wedge f^{-1}(K)$ is fgb- compact set in X, see theorem **(**3.10**)**

Since f is fgb-irresolute function , Then by proposition **(**3.17**)** we have $f(A \wedge f^{-1}(K))$ is fgb-compact set in Y.

Since f is bijective function.

Then $f(A \wedge f^{-1}(K)) = f(A) \wedge K$.

Then $f(A)$ is cfgb- closed set in Y.

$\Leftarrow$ Let $f(A)$ be cfgb-closed set in Y, and let K be fgb-compact set in X

Since f is fgb-irresolute function, then by proposition **(**3.17**)**,we have $f(K)$ is fgb-compact set in Y.

Since $f(A)$ is cfgb-closed set in Y.

Then $f(A) \wedge f(K)$ is fgb-compact set in Y.

Since f is sfgb-compact function , then $f^{-1}(f(A) \wedge f(K))$ is fgb-compact set in X.

Since f is bijective function , then $f^{-1}(f(A) \wedge f(K)) = A \wedge K$.

Then $A \wedge K$ is fgb-compact set in X.

Then A is cfgb-closed set in X □.

Definition 3.23

A fuzzy subset A of X is called compactly fuzzy generalized b-k-closed (in short cfgb-k-closed) set, if $A \wedge K$ is fgb-closed set for every fgb-compact set K in X.

Example 3.24

Every fuzzy subset of a fuzzy discrete space is cfgb-k-closed set .

Proposition 3.25

Every cfgb-k-closed subset of X is cfgb-closed .

Proof :-

Let A be a cfgb-k-closed subset of X, and let K be a fgb- compact set in X.

Then $A \wedge K$ is fgb- closed set .

Since $A \wedge K \leq K$, and K fgb-compact set , then by theorem **(**3.10**)**, we have $A \wedge K$ is fgb-compact set .

Therefore A is cfgb-closed set □.

Theorem 3.26

Every cfgb-closed set in fT_2- space is cfgb-k- closed set .

Proof:-

Let A be cfgb-closed subset of a fT_2- space X, and let K be a fgb- compact set in X.

Then $A \wedge K$ is fgb-compact set .

Since X is fT_2- space , then $A \wedge K$ is fgb- closed set by theorem **(** 3.6**)** .

Then A is cfgb-k-closed □.

Proposition 3.27

Let Y be fT_2- space , and let $f : X \to Y$ be a function, if the only fuzzy subsets of Y which are cfgbk-closed are the whole space and the empty set, and if f is fsgb-compact, and fgb-irresolute function, then f is surjection .

Proof:-

Let $f : X \to Y$ be a fsgb-compact, and fgb-irresolute function.

Let K be a fgb-compact subset of Y.

Since f is a fsgb-compact function , then $f^{-1}(K)$ is fgb-compact set in X.

Since f is fgb-irresolute function, then we have $f(f^{-1}(K))$ is fgb-compact in Y, see proposition **(**3.17**)**.

Since Y is fuzzy T_2-space, then by theorem **(**3.6 **)** ,we have $f(f^{-1}(K))$ is fgb-closed in Y.

But $f(f^{-1}(K)) = f(X \wedge f^{-1}(K)) = f(X) \wedge K$.

Then $f(X) \wedge K$ is fgb-closed set in Y.

Then $f(X)$ is cfgb-k-closed set in Y, but $f(X)$ is not empty set, then $f(X) = Y$.

Therefore $f : X \to Y$ is surjection function □.

References

Ali, H.H. 2014, "On compactly fuzzy b-k-closed sets", Journal of Babylon University, 22(9),pp.1924-1935.

Andrijevic .D,1996," On b-open sets",Math.Vesink,48,pp.59-64.

Benchalli S.S, and Jenifer J. Karnel,2010, "On Fuzzy b- neighborhoods and fuzzy b-Mapping in fuzzy topological spaces ", J.Comp and Math. Sci.Vol.1(6),pp.696-701.

Benchalli S.S, and Jenifer J. Karnel,2010, "On Fuzzy b-open sets in fuzzy topological spaces ", J.Comp and Math. Sci.Vol.1(2),pp. 127-134.

Benchalli S.S, and Jenifer J. Karnel,2011, "On fbg-closed sets and fb-Separation Axioms in fuzzy topological spaces", International Mathematical Forum, 6(51),pp.2547-2559.

Benchalli S.S, and Jenifer J. Karnel,2011, "Weaker and Stronger Forms of Fuzzy b-Irresolute Maps", Int. Journal of Math. Analysis, 5(39),pp.1933-1941..

Chang, C.L. 1968,"fuzzy topological spaces", J.Math.Analysis Appli,24,pp.182-190.

Dang , S.; A.Behra and S.Nanda,1994, "On fuzzy weakly semi – continuous functions", fuzzy sets and systems ,pp.239-245.

Levine, N. 1970, "Generalized closed sets in topology", Rend. Circ. Math. palermo.19,pp.89-96.

M.H.Rashid and D.M.Ali, 2008, " Separation Axioms in maxed fuzzy topological spaces", Bangladesh, J.Acad,Sc.,32(2),211-220.

Mashour, A.S.; Kerre E.E. and M.H.Ghanim, 1984, "Separation axioms, sub spaces and sums in fuzzy topology",Math. Anal, 102, pp.189-202.

Mohammed, H.W. 2011, " On fuzzy regular proper map ", M.Sci.Thesis , Kufa University .

Nouh, A.A. 2005,"On convergent theory in fuzzy topological spaces and its applications", Dml.Cz.math. 55(130),pp.295-316.

Omari A.A. and Noorani M.S.M,2009" On generalized b-closed sets", Bull. Malays. Math. Sci.Soc(2)32,pp.19-30.

Zadeh, L.A. 1965," fuzzy sets",Inform.Contr.,8,pp.338-353.

Zahran, A.M. 1989,"Some Results on Fuzzy Topological Spaces", Ph.D.Thesis.Assiut University.