Some Inequalities under Random C- Condition

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Abstract

In this paper, the notion of random nonexpansive operators is generalized to operators satisfying random $C -$ condition. Some new properties are obtained in uniformly convex Banach spaces, also well we get result of converge on random operator to random point in Banach space.

Keywords: Banach spaces, Random operators, random nonexpansive operators.

1. Introduction and Preliminaries

In 2007, Suzuk [1] introduced generalization of nonexpansive mappings by $C -$ condition to prove some fixed point theorems and convergence theorems. Khan and Suzuki [2] proved a theorem about weak convergence via $C -$ condition in Banach spaces whose dual has the $Kadec - Klee$ property. In this manuscript, we present some of these results in the setting of random operators in uniformly convex separable Banach space. For more results in this area, see the works in [3],[4], [5] and [6]. Now a set Ω, a family $\sum$ of subset of Ω is said to be σ–algebras if its closed complements and countable unions, i.e $\beta \epsilon \sum$ *implies* $\beta^c \epsilon \sum$ and $\beta i \epsilon \sum, i \in N$ implies $\bigcup_i^\infty \beta i \in \sum$, the pair$(\Omega , \sum)$ is called a measurable sets [6].

Definition1.1: [7] Let K be a separable Banach space and δ_n: $\Omega \rightarrow K$ is measurable sequence.

Definition 1.2 : [7] A mapping h : $\Omega \rightarrow K$ is said to be measurable ($\sum -measurable$) if for any open subset V of K,

$$h^{-1}(V) = \{w : h\ (\omega) \cap V \neq \emptyset\} \in \sum$$

Definition 1. 3 : [7] A mapping $h : \ \Omega \times K \ \rightarrow K$ is random operator, if for each fixed $v \in K$ the mapping $h(.,v) : \Omega \ \rightarrow K$ is measurable .

Definition 1.4: [7] A random operator $h: \Omega \times K \to K$ is continuous if $h(v\ ,\ .): \Omega \to K$ is continuous, for each $v \in \Omega$.

Definition 1.5: [7] A measurable mapping $\delta: \Omega \to K$ is random fixed point of a random operator $h: \Omega \times K \to K$ if h(ω , $\delta(\omega)$) = $\delta(\omega)$ For each $\omega \in \Omega$.

Definition 1.6 : [8] A Banach space K is said to be uniformly convex if there exist a strictly increasing function $\eta: [0,2] \to [0,1]$ such that forevery $x, y, p \in K, R > 0$ and $r \in [0\ , 2R]$ The following implication holds:

$$\begin{cases} \|x-p\| \le R \\ \|y-p\| \le R \\ \|x-y\| \ge r \end{cases} \Rightarrow \left\|\frac{x+y}{2} - p\right\| \le \left(1 - \eta\left(\frac{r}{R}\right)\right).R$$

Definition : 1.7 : [9] Let C be a subset of Banach space K, A mapping $h : C \to K$ is said to be demiclosed at $u \in K$ if for any sequence $\{v_n\}$ in C with v_n converges weakly to v and $hv_n \to u$, it follows that $v \in C$ and $hv = u$

Lemma 1.8 : [10] Let$K$ be a uniformly convex Banach space , 0$< p \le t_n \le q < 1$. For all $n \in N$ $suppose\ that$ $\{v\}$and $\{u_n\}$ are two sequences of K such thate $\limsup_{n\to\infty} \|v_n\| \le r$, $\limsup_{n\to\infty} \|u_n\| \le r$ and

$\lim_{n\to\infty} \|t_n v_n + (1-t_n)u_n\| = r$ for some $r \ge 0$ Then $\lim_{n\to\infty} \|v_n - u_n\| = 0$.

Lemma 1.9 : [2] Let K be a uniformly convex Banach space. Let $\{u_n\}, \{v_n\}$and $\{z_n\}$ be sequences in K let d and t be real numbers with $d \in (0\ , \infty)$ and $t \in (0,1)$ Assume that

$\lim_{n\to\infty} \|u_n - v_n\| = d$, $\lim_{n\to\infty} \sup \|u_n - z_n\| \le (1-t)\, d$ and

$\lim_{n\to\infty} \sup \|v_n + z\|_n < td$, then :

$$\lim_{n\to\infty} \|tu_n + (1-t)v_n - z_n\| = 0.$$

2. Main Results

In this section we need the Banach space is separable and the following concept and properties for random condition (C)

Definition 2.1 (condition random C)

Let K be a subset of separable Banach space K. Let $G: \Omega \times K \to K$ be a random operator. Then G is said to be satisfay the random condition C (RC) if :

$$\frac{1}{2}\|v - G(\omega, v)\| \le \|v - u\| \ implies\ \|G(\omega, v) - G\ (\omega, u)\| \le \|v - u\|$$

For all $v, u \in K$ and for all $\omega \in \Omega$

Proposition 2.2 Let $G: \Omega \times C \to C$ be a measurable mapping where K a subset of a separable Banach space K. Assume that G satisfies condition (RC) , for all $v, u \in C$ and for all $\omega \in \Omega$, then te following hold:

i) $\|G(\omega, v) - G^2(\omega, v)\| \leq \|v - G(\omega, v)\|$

ii) Either $\frac{1}{2}\|v - G(\omega, v)\| \leq \|v - u\|$
or $\frac{1}{2}\|G(\omega, v) - G^2(\omega, v)\| \leq \|G(\omega, v) - u\|$ *holds*

iii) Either $\|G(\omega, v) - G(\omega, u)\| \leq \|v - u\|$
or $\|G^2(\omega, v) - G(\omega, u)\| \leq \|G(\omega, v) - u\|$ holds

Proof:

i) Follows from $\frac{1}{2}\|v - G(\omega, v)\| \leq \|v - G(\omega, v)\|$and by the condition (RC) then we have :

$$\|G(\omega, v) - G^2(\omega, v)\| \leq \|v - G(\omega, v)\|$$

ii) Arguing by contradiction we assume that
$\frac{1}{2}\|v - G(\omega, u)\| > \|v - u\|$ and

$$\frac{1}{2}\|G(\omega, v) - G^2(\omega, v)\| > \|G(\omega, v) - u\|$$

Then we have by (i)

$$\begin{aligned}\|v - G(\omega, v)\| &\leq \|v - u\| + \|G(\omega, v) - u\| \\ &< \frac{1}{2}\|v - G(\omega, v)\| + \frac{1}{2}\|G(\omega, v) - G^2(\omega, v)\| \\ &\leq \frac{1}{2}\|v - G(\omega, v)\| + \frac{1}{2}\|v - G(\omega, v)\| \\ &= \|v - G(\omega, v)\|\end{aligned}$$

This is contradiction

iii) Follows from (ii)
Either $\frac{1}{2}\|v - G(\omega, v)\| \leq \|v - u\|$ and by condition (RC) then implies.

$$\|G(\omega, v) - G(\omega, u)\| \leq \|v - u\|$$

$$or \quad \frac{1}{2}\|G(\omega, v) - G^2(\omega, u)\| \leq \|G(\omega, v) - u\|$$

and by condition (RC) then implies :

$$\|G^2(\omega, v) - G(\omega, v)\| \leq \|G(\omega, v) - u\|.$$

Proposion 2.3 Let C be a subset of a separable Banach space K Let $G: \Omega \times C \to C$ *be* ameasurable

mapping. Assume that G satisfies condition (RC) for all $v, u \in K$and for all $\omega \in \Omega$ then :

i) $\|v - G(\omega, u)\| \leq 3\|G(\omega, v) - v\| + \|v - u\|$

ii) $\|u - G(\omega, u)\| \leq 3\|G(\omega, v) - v\| + 2\|v - u\|$

By Proposition 2.2 (iii)

Either $\|G(\omega, v) - G(\omega, u)\| \leq \|v - u\|$

or $\|G^2(\omega, v) - G(\omega, u)\| \leq \|G(\omega, v) - u\|$ holds

In the first case, we have :

$$\|v - G(\omega, u)\| \leq \|v - G(\omega, v)\| + \|G(\omega, v) - G(\omega, u)\|$$
$$\leq \|v - G(w, v)\| + \|v - u\|$$
$$\therefore \|v - G(\omega, u)\| \leq \|v - G(\omega, v)\| + \|v - u\|$$

In the second case, we have by **Proposition2.2 (i)**

$$\|v - G(\omega, u)\| \leq \|v - G(\omega, v)\| + \|G(\omega, v) - G^2(\omega, v)\| + \|G^2(\omega, v) - G(\omega, u)\|$$

$$\leq \|v - G(\omega, v)\| + \|v - G(\omega, v)\| + \|G(\omega, v) - u\|$$
$$\leq 2\|v - G(\omega, v)\| + \|G(\omega, v) - u\|$$
$$\leq 2\|v - G(\omega, v)\| + \|G(\omega, v) - v\| + \|v - u\|$$
$$= 2\|v - G(\omega, v)\| + \|v - G(\omega, v)\| + \|v - u\|$$
$$\leq 3\|v - G(\omega, v)\| + \|v - u\|$$

Now:

$$\|u - G(\omega, u)\| \leq \|u - v\| + \|v - G(\omega, u)\|$$

By using (i)

$$\leq \|u - v\| + 3\|v - G(\omega, v)\| + \|v - u\|$$
$$\therefore \|u - G(\omega, u)\| \leq 3\|v - G(\omega, v)\| + 2\|v - u\|$$

Proposition 2.4 Let C be a bounded and convex subset of uniformly convex separable Banach space K. let $G: \Omega \times C \to C$ be arandom operator. Assume that G satisfies condition (RC).Then for any $\in > 0$ there exist $\eta(\in) > 0$ such that for any $t \in [0,1]$ and for any $u, v \in C$ with :

$\|G(\omega, v) - v\| < \eta(\in)$, $\|G(\omega, u) - u\| < \eta(\in)$ we have :

$\|G(\omega, tv + (1 - t)u) - (tv + (1 - t)u\| < \in$

Proof:

Arguing by contradiction we assume that there exist $\in > 0$,sequences $\{v_n\}$,$\{u_n\}$ in C , $t_n \in [0,1]$ and

$$\|G(\omega, v_n) - v_n\| < \frac{1}{n} \quad , \quad \|G(\omega, u_n) - u_n\| < \frac{1}{n}$$

And

$$\|G(\omega, t_n v_n + (1-t_n)u_n) - (t_n v_n \ + (1-t_n)u_n\| \geq \in$$

Denote $z_n = t_n v_n + (1 - t_n)$ and $G(\omega, z_n)$ using

Proposition 2.3 (ii) we have that

$$\begin{aligned} 0 <\in &\leq \lim_{n\to\infty} inf \|z_n - G(\omega, z_n)\| \\ &\leq \lim_{n\to\infty} inf\, (3\|u_n - G(\omega, u_n)\| + 2\|z_n - u_n\|) \\ &< \lim_{n\to\infty} inf \left(3\left(\frac{1}{n}\right) + 2\|z_n - u_n\|\right) \\ &= 2\lim_{n\to\infty} inf \|z_n - u_n\| \end{aligned}$$

Similarly we can show that $0 < \lim_{n\to\infty} inf \|z_n - v_n\|$

Thus $0 < \lim_{n\to\infty} inf \|u_n - v_n\|$ .Since C is bounded and

$$\begin{aligned} 0 < \lim_{n\to\infty} inf \|z_n - v_n\| &= \lim_{n\to\infty} inf\, t_n \|u_n - v_n\| \\ &\leq \lim_{n\to\infty} inf\, t_n \ . \ \sup\|u_n - v_n\| \end{aligned}$$

We have that $0 < \lim_{n\to\infty} \inf t_n$, similarity we can prove that Lim sup $t_n < 1$. So without loss of generality. We may as sume that $\{\|u_n - v_n\|\}$ and $\{t_n\}$ converge to some real number.

$d \in (0,\infty)$ and $t \in (0,1)$, respectively .

Since $\lim_{n\to\infty}\|u_n - G(\omega, u_n)\| = 0$ and $0 < \lim_{n\to\infty} inf \|u_n - z_n\|$ we have that $\frac{1}{2}\|u_n - G(\omega, u_n)\| \leq \|u_n - z_n\|$ for sufficiently Large $n \in N$. From

condition (RC) we get that :

$$\|G(\omega, u_n) - G(\omega, z_n)\| \leq \|u_n - z_n\|$$

Similarly we can show

$$\|G(\omega, u_n) - G(\omega, z_n)\| \leq \|v_n - z_n\|$$

Then now: Let $G(\omega , z_n) = x_n$

$\lim_{n\to\infty} sup\|v_n - x_n\| \leq$

$$\begin{aligned} &\lim_{n\to\infty} sup(\|v_n - G(\omega, v_n)\| + \|G(\omega, v_n) - G(\omega, x_n\|) \\ &\leq \lim_{n\to\infty}(\|v_n - G(\omega, v_n\| + \|v_n - z_n\|) \\ &= (1-t)d \end{aligned}$$

From **Lemma (1.9)** we obtain :

$0 <\in< \lim_{n\to\infty}\|z_n - x_n\|$

$$\leq \lim_{n\to\infty}(\|z_n - (tu_n + (1-t)v_n)\| + \|(tu_n + (1-t)v_n) - x_n\|)$$

$0 < 0$ which is a contradiction.

Proposition 2.5 Let C be abounded and convex subset of a uniformly convex separable Banach space K. Let $G: \Omega \times C \to C$ be a random operator. Assume G satisfies condition (RC). Then I-G is demiclosed at zero. That is if $\{v_n\}$ in C converges weakly to $z \in C$ and $\lim_{n\to\infty}\|G(\omega, v_n) - v_n\| = 0$, then : $G(\omega, z) = z$

Proof:

Let η be a function from $(0, \infty)$ in to it self which satisfies the conclution of **proposition (2.4)** we assume that $\{ v_n\}$ converge weakly to z and $\lim_{n\to\infty}\|G(\omega, v_n) - v_n\| = 0$ let $\in > 0$ be arbitrary chosen. Define a strictly decreasing sequence $\{\in_n\}$ in $(0, \infty)$ by $\in_1 = \in$ and $\in_{n+1} = \min\{\in_n, \eta(\in_n)\}/2$

It is obvious that $\in_{n+1} < \eta(\in_n)$. Choose a sub sequences $\{ v_{ni}\}$ of $\{v_n\}$ such that ,

$\|v_{ni} - G(\omega, v_{ni})\| < \eta(\in_n). since\{v_n i\}$ converges weakly to z , v_n belongs to the closed convex hull of $\{v_{ni}: n \in N\}$ so there exist $u \in C$and $p \in N$ such that $\|u - z\| < \in$ and u belongs to the convex hull of $\{v_{ni}$: n =1 ,2,3,…..,p}

From **Proposition (2.4)** we get that

$\|G(w, u) - u\| < \in$. So we have from **Proposition (2.3)** that :

$$\|G(\omega, z) - z\| \leq 3\|G(\omega, u) - u\| + 2\|u - z\| \leq 5 \in$$

Since $\in > 0$ is arbitrary. We obtain $G(\omega, z) = z$.

Theorem (2. 6)

Let C be a nonempty closed convex subset of a uniformly convex Banach space K. Let G and h be two random operators $G, h : \Omega\times C \to C$ satisfying condition (RC) such that :

$$\left\{\begin{array}{ll} \delta_1: \Omega \to C, & \bar{\delta}_1 : \Omega \to C \\ \delta_{n+1} = (1 - a_n)G(\omega, \delta_n) + a_n h(\omega, \overline{\delta}_n) & \\ \overline{\delta}_n = (1 - b_n)\delta_n + b_n\, G(\omega, \delta_n) & \end{array}\right\} \quad (2.1)$$

Where $\{a_n\}$ and $\{b_n\}$ are sequences in $[\varepsilon, 1 - \varepsilon] n \in N$. And for some ε in $(0 , 1)$if $RF = RF(G) \cap RF(h) \neq \emptyset$ then $\lim_{n\to\infty}\|\delta_n(\omega) - \mathfrak{z}(\omega)\|$

exists and $\lim_{n\to\infty}\|\delta_n(\omega) - G(\omega, \delta_n)\| = 0 = \lim_{n\to\infty}\|\delta_n - h(\omega, \delta_n)\|$

Proof :

Let $\mathfrak{z}(\omega) \epsilon RF$. By use of condition (RC) , we get : in $(0 , 1)$

$\frac{1}{2}\|\mathfrak{z}(\omega) - G(\omega, \mathfrak{z}(\omega))\| = 0 \leq \|\delta_n(\omega) - \mathfrak{z}(\omega)\| \Longrightarrow$

$$\left\|G\left(\omega,\delta_n(\omega)\right)-G\left(\omega,\mathfrak{z}(\omega)\right)\right\|\le\|\delta_n(\omega)-\mathfrak{z}(\omega)\| \qquad (2.2)$$

$\frac{1}{2}\left\|\mathfrak{z}(\omega)-G\left(\omega,\mathfrak{z}(\omega)\right)\right\|=0\le\left\|\overline{\delta}_n(\omega)-\mathfrak{z}(\omega)\right\|\Longrightarrow$

$$\left\|h\left(\omega,\overline{\delta}_n(\omega)\right)-h\left(\omega,\mathfrak{z}(\omega)\right)\right\|\le\left\|\overline{\delta}_n(\omega)-\mathfrak{z}(\omega)\right\| \ldots\ldots\ldots\ldots\ldots (2.3)$$

Using inequalities (2.2) and (2.3) a long with (2.1), we have :

$$\|\delta_{n+1}(\omega)-\mathfrak{z}(\omega)\|=\left\|(1-a_n)\left(G\left(\omega,\delta_n(\omega)\right)-\mathfrak{z}(\omega)\right)+a_n\left(h\left(\omega,\bar{\delta}_n(\omega)\right)-\mathfrak{z}(\omega)\right)\right\|$$

$$\le(1-a_n)\left\|G\left(\omega,\delta_n(\omega)\right)-\mathfrak{z}(\omega)\right\|+a_n\left\|h\left(\omega,\bar{\delta}_n(\omega)\right)-\mathfrak{z}(\omega)\right\|$$
$$\le(1-a_n)\|\delta_n(\omega)-\mathfrak{z}(\omega)\|+a_n\left\|\bar{\delta}_n(\omega)-\mathfrak{z}(\omega)\right\|$$

$$=(1-a_n)\|\delta_n(\omega)-\mathfrak{z}(\omega)\|+a_n\left\|(1-b_n)\delta_n(\omega)+b_nG\left(\omega,\delta_n(\omega)\right)-\mathfrak{z}(\omega)\right\|$$

$$\le(1-a_n)\|\delta_n(\omega)-\mathfrak{z}(\omega)\|+a_n(1-b_n)\|\delta_n(\omega)-\mathfrak{z}(\omega)\|+a_n b_n\left\|G\left(\omega,\delta_n(\omega)\right)-\mathfrak{z}(\omega)\right\|$$

$$\le(1-a_n)\|\delta_n(\omega)-\mathfrak{z}(\omega)\|+a_n(1-b_n)\|\delta_n(\omega)-\mathfrak{z}(\omega)\|$$
$$+a_n b_n\|\delta_n(\omega)-\mathfrak{z}(\omega)\|$$
$$=\|\delta_n(\omega)-\mathfrak{z}(\omega)\|$$

Therefore $\lim_{n\to\infty}\|\delta_n(\omega)-\mathfrak{z}(\omega)\|$ exist for any $\mathfrak{z}(\omega)\in RF$

Let $\lim_{n\to\infty}\|\delta_n(\omega)-\mathfrak{z}(\omega)\|=a.$ Consider

$$\left\|\bar{\delta}_n(\omega)-\mathfrak{z}(\omega)\right\|=\left\|b_nG\left(\omega,\delta_n(\omega)\right)+(1-b_n)\delta_n(\omega)-\mathfrak{z}(\omega)\right\|$$
$$\le b_n\left\|G\left(\omega,\delta_n(\omega)\right)-\mathfrak{z}(\omega)\right\|+(1-b_n)\|\delta_n(\omega)-\mathfrak{z}(\omega)\|$$
$$\le b_n\|\delta_n(\omega)-\mathfrak{z}(\omega)\|+(1-b_n)\|\delta_n(\omega)-\mathfrak{z}(\omega)\|$$
$$=\|\delta_n(\omega)-\mathfrak{z}(\omega)\|,$$

which implies that

$$\lim_{n\to\infty}\sup\left\|\bar{\delta}_n(\omega)-\mathfrak{z}(\omega)\right\|\le a$$

Using (2.2) and (2.3) , we have

$\lim_{n\to\infty}\sup\left\|G\left(\omega,\delta_n(\omega)\right)-\mathfrak{z}(\omega)\right\|\le a$ and

$\lim_{n\to\infty}\sup\left\|h\left(\omega,\bar{\delta}_n(\omega)\right)-\mathfrak{z}(\omega)\right\|\le a. \qquad (2.4)$

Moreover, we have

$$a = \lim_{n\to\infty} \|\delta_{n+1}(\omega) - \mathfrak{z}(\omega)\|$$
$$= \lim_{n\to\infty} \left\|(1-a_n)\left(G(\omega,\delta_n(\omega)) - \mathfrak{z}(\omega)\right) + a_n\left(h\left(\omega,\bar{\delta}_n(\omega)\right) - \mathfrak{z}(\omega)\right)\right\| \qquad (2.5)$$

Therefore, by using (2.4) , (2.5) and **Lemma (1.8)** we have

$$\lim_{n\to\infty} sup \left\|G(\omega,\delta_n(\omega)) - h\left(\omega,\bar{\delta}_n(\omega)\right)\right\| = 0 \qquad (2.6)$$

On the other hand, we have

$$\|\delta_{n+1}(\omega) - \mathfrak{z}(\omega)\| = \|(1-a_n)G(\omega,\delta_n(\omega)) + a_n h(\omega,\delta_n(\omega)) - \mathfrak{z}(\omega)\|$$
$$= \left\|\left(G(\omega,\delta_n(\omega)) - \mathfrak{z}(\omega)\right) + a_n\left(h\left(\omega,\bar{\delta}_n(\omega)\right) - G(\omega,\delta_n(\omega))\right)\right\|$$
$$\leq \|G(\omega,\delta_n(\omega)) - \mathfrak{z}(\omega)\| + a_n\left\|G(\omega,\delta_n(\omega)) - h\left(\omega,\bar{\delta}_n(\omega)\right)\right\|$$

Taking $Lim\ inf$ on both the sides, we get

$$a \leq \lim_{n\to\infty} inf \ \|G(\omega,\delta_n(\omega)) - \mathfrak{z}(\omega)\|$$

Which implies with (2.4) that $\|G(\omega,\delta_n(\omega)) - \mathfrak{z}(\omega)\| = a.$ (2.7)

Using (2.7) , we have

$$\|G(\omega,\delta_n(\omega)) - \mathfrak{z}(\omega)\|$$
$$\leq \left\|G(\omega,\delta_n(\omega)) - h\left(\omega,\bar{\delta}_n(\omega)\right)\right\| + \left\|h\left(\omega,\bar{\delta}_n(\omega)\right) - \mathfrak{z}(\omega)\right\|$$
$$\leq \left\|G(\omega,\delta_n(\omega)) - h\left(\omega,\bar{\delta}_n(\omega)\right)\right\| + \|\bar{\delta}_n(\omega) - \mathfrak{z}(\omega)\|$$

Taking $Lim\ inf$ on be both sides and using (2.7), we find that

$$a \leq \lim_{n\to\infty} inf \ \|\bar{\delta}_n(\omega) - \mathfrak{z}(\omega)\| \qquad (2.8)$$

Hence, by (2.3) and (2.8), we have

$$\lim_{n\to\infty} \|\bar{\delta}_n(\omega) - \mathfrak{z}(\omega)\| = a \qquad (2.9)$$

Since $a = \lim_{n\to\infty} \|\bar{\delta}_n(\omega) - \mathfrak{z}(\omega)\|$

$$= \lim_{n\to\infty} \left\|(1-b_n)(\delta_n(\omega) - \mathfrak{z}(\omega)) + b_n\left(G(\omega,\delta_n(\omega))\right) - \mathfrak{z}(\omega)\right\|$$

We find from **Lemma (1.8)** that

$$\lim_{n\to\infty} \|G(\omega,\delta_n(\omega)) - \delta_n(\omega)\| = 0 \qquad (2.10)$$

Since $\|\bar{\delta}_n(\omega) - \delta_n(\omega)\| = \|b_n G(\omega,\delta_n(\omega)) + (1-b_n)\delta_n(\omega) - \delta_n(\omega)\|$

$$= \left\|G\left(\omega, \delta_n(\omega)\right) - \delta_n(\omega)\right\|$$

Making use of (2.10) , we get :

$$\lim_{n\to\infty}\left\|\bar{\delta}_n(\omega) - \delta_n(\omega)\right\| = 0 \qquad (2.11)$$

Using (2.6), (2.10) , (2.11) and **Proposition (2.2) (ii)**, we have

$$\left\|\delta_n(\omega) - h\left(\omega, \delta_n(\omega)\right)\right\| \leq 3\left\|\bar{\delta}_n(\omega) - h\left(\omega, \bar{\delta}_n(\omega)\right)\right\| + 2\left\|\delta_n(\omega) - \bar{\delta}_n(\omega)\right\|$$
$$\leq 3\left\|\bar{\delta}_n(\omega) - G\left(\omega, \delta_n(\omega)\right)\right\| + 3\left\|G\left(\omega, \delta_n(\omega)\right) - h\left(\omega, \bar{\delta}_n(\omega)\right)\right\| + 2\left\|\delta_n(\omega) - \bar{\delta}_n(\omega)\right\|$$
$$= 3\left\|(1 - b_n)\left(\delta_n(\omega)\right) + b_n G\left(\omega, \delta_n(\omega)\right) - G\left(\omega, \delta_n(\omega)\right)\right\| + 3\left\|G\left(\omega, \delta_n(\omega)\right) - h\left(\omega, \bar{\delta}_n(\omega)\right)\right\| + 2\left\|\delta_n(\omega) - \bar{\delta}_n(\omega)\right\|$$
$$= 3(1 - b_n)\left\|\delta_n(\omega) - G\left(\omega, \delta_n(\omega)\right)\right\| + 3\left\|G\left(\omega, \delta_n(\omega)\right) - h\left(\omega, \bar{\delta}_n(\omega)\right)\right\| + 2\left\|\delta_n(\omega) - \bar{\delta}_n(\omega)\right\|$$

Yielding there by Lim$\left\|h\left(\omega, \delta_n(\omega)\right) - \delta_n(\omega)\right\| = 0$

This concludes the proof.

Finally, we suggest using recent results to generalized the results in [11] or study of these properties for in other spaces, such as, in modular spaces [12].

Conflict of Interests.

There are non-conflicts of interest .

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الخلاصة

في هذا البحث تطرقنا الى تعميم للتطبيق المتمدد الذي يحقق الشرط العشوائي C .بعض الخواص الجديدة حصلنا عليها في فضاء بناخ الذي يحمل صفة التحدب العام كذلك نحن نحصل على نتيجة للتقارب حول التطبيق العشوائي الى نقطة عشوائية في فضاء بناخ .