11- Modular Characters for $\overline{S}_{27}$

Ahmed Hussein Jassim

Department of Mathematics, College of Science / University of Basrah, Iraq
ahmedhussein1981@ymail.com

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Abstract

In this paper we find decomposition matrices for the spin characters $\bar{S}_{27}$ modulo 11 by using method of $(r, \bar{r})$-inducing.
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I. Introduction

The projective characters of symmetric group S_n it's called the spin characters, which are ordinary characters of $\bar{S}_n$ indexed by the bar partitions of n[1],[2]. The information about the spin characters is little and There is no general method for finding the decomposition matrix for spin characters of S_n that is the relation between the ordinary and modular characters [3],[4]. Some who searched the same topic when $p = 7$, like A.O. Morris, A.K.Yaseen[4], *S. A.* Taban [9], A. H. Jassim, M. M. Jawad [5],[7]. Some of the symbols used in this paper, $(<\lambda>)^{no}$ means no the number of i.m.s. in $<\lambda>$, (i.m.s.) is irreducible modular spin character. (p.i.s.) is principle indecomposable spin character [7].

II. Results

In this paper we found a decomposition matrices for spin S_{27} modulo 11 was a degree (288,241) [4],[6], where it contains are 63 blocks, B_1, B_2, B_3 of defect two and, $B_4, B_5, \dots, B_{17}$ are of defect one. The others blocks of defect zero.

Lemma (1). The Brauer trees for the blocks $B_4, B_5, \dots, B_{17}$ respectively are:

$$\langle 24,2,1\rangle^* \_\_ \langle 13,12,2\rangle^* \_\_ \langle 13,11,2,1\rangle = \langle 13,11,2,1\rangle' \_\_ \langle 13,8,3,2,1\rangle^* \_\_ \langle 13,7,4,2,1\rangle^* \_\_ \langle 13,6,5,2,1\rangle^*,$$

$$\langle 23,3,1\rangle^* \_\_ \langle 14,12,1\rangle^* \_\_ \langle 12,11,3,1\rangle = \langle 12,11,3,1\rangle' \_\_ \langle 12,9,3,2,1\rangle^* \_\_ \langle 12,7,4,3,1\rangle^* \_\_ \langle 12,6,5,3,1\rangle^*,$$

$$\begin{array}{ccccc} \langle 21,6\rangle \_\_ \langle 17,10\rangle & \backslash & & / & \langle 10,9,6,2\rangle \_\_ \langle 10,8,6,3\rangle \_\_ \langle 10,7,6,4\rangle \\ & & \langle 11,10,6\rangle^* & & \\ \langle 21,6\rangle' \_\_ \langle 17,10\rangle' & / & & \backslash & \langle 10,9,6,2\rangle' \_\_ \langle 10,8,6,3\rangle' \_\_ \langle 10,7,6,4\rangle' \end{array},$$

$$\langle 21,4,2\rangle^* \_\_ \langle 15,10,2\rangle^* \_\_ \langle 13,10,4\rangle^* \_\_ \langle 11,10,4,2\rangle = \langle 11,10,4,2\rangle' \_\_ \langle 10,8,4,3,2\rangle^* \_\_ \langle 10,6,5,4,2\rangle^*,$$

$$\begin{matrix}\langle 20,7\rangle\_\_\langle 18,9\rangle \\ \langle 20,7\rangle'\_\_\langle 18,9\rangle'\end{matrix} \rangle \langle 11,9,7\rangle^{*} \langle \begin{matrix}\langle 10,9,7,1\rangle\_\_\langle 9,8,7,3\rangle\_\_\langle 9,7,6,5\rangle \\ \langle 10,9,7,1\rangle'\_\_\langle 9,8,7,3\rangle'\_\_\langle 9,7,6,5\rangle'\end{matrix},$$

$\langle 20,6,1\rangle^{*}\_\_\langle 17,9,1\rangle^{*}\_\_\langle 12,9,6\rangle^{*}\_\_\langle 11,9,6,1\rangle = \langle 11,9,6,1\rangle'\_\_\langle 9,8,6,3,1\rangle^{*}\_\_\langle 9,7,6,4,1\rangle^{*}$,

$\langle 20,4,3\rangle^{*}\_\_\langle 15,9,3\rangle^{*}\_\_\langle 14,9,4\rangle^{*}\_\_\langle 11,9,4,3\rangle = \langle 11,9,4,3\rangle'\_\_\langle 10,9,4,3,1\rangle^{*}\_\_\langle 9,6,5,4,3\rangle^{*}$,

$\langle 19,7,1\rangle^{*}\_\_\langle 18,8,1\rangle^{*}\_\_\langle 12,8,7\rangle^{*}\_\_\langle 11,8,7,1\rangle = \langle 11,8,7,1\rangle'\_\_\langle 9,8,7,2,1\rangle^{*}\_\_\langle 8,7,6,5,1\rangle^{*}$,

$\langle 19,6,2\rangle^{*}\_\_\langle 17,8,2\rangle^{*}\_\_\langle 13,8,6\rangle^{*}\_\_\langle 11,8,6,2\rangle = \langle 11,8,6,2\rangle'\_\_\langle 10,8,6,2,1\rangle^{*}\_\_\langle 8,7,6,4,2\rangle^{*}$,

$$\begin{matrix}\langle 19,5,2,1\rangle\_\_\langle 16,8,2,1\rangle\_\_\langle 13,8,5,1\rangle\_\_\langle 12,8,5,2\rangle \\ \langle 19,5,2,1\rangle'\_\_\langle 16,8,2,1\rangle'\_\_\langle 13,8,5,1\rangle'\_\_\langle 12,8,5,2\rangle'\end{matrix} \rangle \langle 11,8,5,2,1\rangle^{*} \langle \begin{matrix}\langle 8,7,5,4,2,1\rangle \\ \langle 8,7,5,4,2,1\rangle'\end{matrix}$$

$\langle 18,6,3\rangle^{*}\_\_\langle 17,7,3\rangle^{*}\_\_\langle 14,7,6\rangle^{*}\_\_\langle 11,7,6,3\rangle = \langle 11,7,6,3\rangle'\_\_\langle 10,7,6,3,1\rangle^{*}\_\_\langle 9,7,6,3,2\rangle^{*}$,

$$\begin{matrix}\langle 18,6,2,1\rangle\_\_\langle 17,7,2,1\rangle\_\_\langle 13,7,6,1\rangle\_\_\langle 12,7,6,2\rangle \\ \langle 18,6,2,1\rangle'\_\_\langle 17,7,2,1\rangle'\_\_\langle 13,7,6,1\rangle'\_\_\langle 12,7,6,2\rangle'\end{matrix} \rangle \langle 11,7,6,2,1\rangle^{*} \langle \begin{matrix}\langle 8,7,6,3,2,1\rangle \\ \langle 8,7,6,3,2,1\rangle'\end{matrix},$$

$$\begin{matrix}\langle 18,5,3,1\rangle\_\_\langle 16,7,3,1\rangle\_\_\langle 14,7,5,1\rangle\_\_\langle 12,7,5,3\rangle \\ \langle 18,5,3,1\rangle'\_\_\langle 16,7,3,1\rangle'\_\_\langle 14,7,5,1\rangle'\_\_\langle 12,7,5,3\rangle'\end{matrix} \rangle \langle 11,7,5,3,1\rangle^{*} \langle \begin{matrix}\langle 9,7,5,3,2,1\rangle \\ \langle 9,7,5,3,2,1\rangle'\end{matrix},$$

$\langle 17,4,3,2,1\rangle^{*}\_\_\langle 15,6,3,2,1\rangle^{*}\_\_\langle 14,6,4,2,1\rangle^{*}\_\_\langle 13,6,4,3,1\rangle^{*}\_\_\langle 12,6,4,3,2\rangle^{*}\_\_\langle 11,6,4,3,2,1\rangle = \langle 11,6,4,3,2,1\rangle'$

Proof. First we find B_6 by used the $(6,6) -$ inducing p.i.s. $D_{76}, D_{77}, \dots, D_{80}$, of S_{26} to S_{27} we get on $k_1, k_2, \dots, k_5$. Since it's associate[4] so $\langle 22,6\rangle \neq \langle 22,6\rangle'$ then k_1 splits or there are two columns: $Y_1 = a_1\langle 21,6\rangle + a_2\langle 17,10\rangle + a_3\langle 11,10,6\rangle^{*} + a_4\langle 10,9,6,2\rangle + a_5\langle 10,8,6,3\rangle + a_6\langle 10,7,6,4\rangle$, $Y_2 = a_1\langle 21,6\rangle' + a_2\langle 17,10\rangle' + a_3\langle 11,10,6\rangle^{*} + a_4\langle 10,9,6,2\rangle' + a_5\langle 10,8,6,3\rangle' + a_6\langle 10,7,6,4\rangle'$, $a_1, a_2, \dots, a_6 \in \{0,1\}$($B_6$of defect one)[1]. Let $a_1 = 1$(if $a_1 = 0$ contradiction) as $\langle 21,6\rangle \downarrow S_{26} \cap \langle 10,9,6,2\rangle \downarrow S_{26}$ has no i.m.s so $a_4 = 0$. In same way we get $a_5, a_6 = 0$. But deg$Y_1, Y_2 \equiv 0 \ mod 11^2$ only when $a_1 = a_2 = 1$, $a_3 = 0$ so that $k_1 = d_{111} + d_{112}$. In the same way we part k_4, k_5 to $d_{117}, d_{118}, d_{119}, d_{120}$. Since B_6 of defect one so k_2, k_3 are splits to $d_{113}, d_{114}, d_{115}, d_{116}$ [6]. Block B_8 can be found by used the same way above.

For B_{13} we used $(r, \bar{r})$-inducing p.i.s. $D_{121}, D_{122}, \dots, D_{124}, D_{133}, D_{127}, D_{129}, D_{130}$, of S_{26} to S_{27} we got on $d_{156}, d_{157}, d_{158}, d_{159}, k_1, k_2, d_{164}, d_{165}$. Since it's associate so k_1 divided or there are two columns:$Y_1 = a_1\langle 13,8,5,1\rangle + a_2\langle 12,8,5,2\rangle + a_3\langle 11,8,5,2,1\rangle^{*} + a_4\langle 8,7,5,4,2,1\rangle$, $Y_2 = a_1\langle 13,8,5,1\rangle' + a_2\langle 12,8,5,2\rangle' + a_3\langle 11,8,5,2,1\rangle^{*} + a_4\langle 8,7,5,4,2,1\rangle'$,$a_1, a_2, \dots, a_4 \in \{0,1\}$[6]. Let $a_1 = 1$ and, since $\langle 13,8,5,1\rangle \downarrow S_{26} \cap \langle 11,8,5,2,1\rangle^{*} \downarrow S_{26}$ has no i.m.s so $a_3 = 0$, the same we get $a_4 = 0$ so deg$Y_1, Y_2 \equiv 0 mod\ 11^2$ only when $a_1, a_2 = 1, a_3 = 0$ then $k_1 = d_{160} + d_{161}$. Since $\langle 12,8,5,2,1\rangle$, $\langle 12,8,5,2,1\rangle'$ are p.i.s. of S_{28} (of defect zero in $S_{28}, p = 11$)and,

$\langle 12,8,5,2,1\rangle \downarrow_{(1,0)} S_{28} = \langle 11,8,5,2,1\rangle^{*} + \langle 12,8,5,2\rangle = d_{162}$, $\langle 12,8,5,2,1\rangle' \downarrow_{(1,0)} S_{28} = \langle 11,8,5,2,1\rangle^{*} + \langle 12,8,5,2\rangle' = d_{163}$ then $k_2 = d_{162} + d_{163}$. In the same way we proved the blocks B_{15}, B_{16}. The other blocks we found were used the $(r,\bar{r})-$inducing from S_{26} to S_{27} directly. Finally we find the Brauer trees from the blocks of the decomposition [1].

Lemma (2). The block B_1 is of a double and the decomposition matrix for is table (1).

Table (1)

$\langle 27\rangle^{*}$	1																			
$\langle 22,5\rangle$	1	1																		
$\langle 21,5,1\rangle^{*}$		1	1																	
$\langle 20,5,2\rangle^{*}$			1	1																
$\langle 9,5,3\rangle^{*}$				1	1															
$\langle 18,5,4\rangle^{*}$					1	1														
$\langle 16,11\rangle$	1	1					1													
$\langle 16,10,1\rangle^{*}$	2	1	1				1	1												
$\langle 16,9,2\rangle^{*}$			1	1				1	1											
$\langle 16,8,3\rangle^{*}$				1	1				1	1										
$\langle 16,7,4\rangle^{*}$					1	1				1	1									
$\langle 16,6,5\rangle^{*}$						1					1									
$\langle 15,7,5\rangle^{*}$										1	1	1								
$\langle 14,8,5\rangle^{*}$									1	1		1	1							
$\langle 13,9,5\rangle^{*}$								1	1				1	1						
$\langle 12,10,5\rangle^{*}$	2						2	1						1	2					
$\langle 11,10,5,1\rangle$							1							1	1	1				
$\langle 11,9,5,2\rangle$													1	1	1	1	1			
$\langle 11,8,5,3\rangle$												1	1				1	1		
$\langle 11,7,5,4\rangle$												1						1		
$\langle 10,9,5,2,1\rangle^{*}$															1		1		1	
$\langle 10,8,5,3,1\rangle^{*}$																1	1	1	1	1
$\langle 10,7,5,4,1\rangle^{*}$																		1		1
$\langle 9,8,5,3,2\rangle^{*}$																1				1
$\langle 9,7,5,4,2\rangle^{*}$																			1	1
$\langle 8,7,5,4,3\rangle^{*}$																			1	
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}

Proof. We found the table above through using $(\mathrm{r},\bar{r})-$inducing of p.i.s. $D_1, D_3, D_5, D_7, D_{146}, D_{11}, D_{15}, D_{17}, D_{19}, D_{21}, D_{13}, D_{150}, D_{25}, D_{27}, \dots, D_{39}$ of S_{26} to S_{27} get on $d_1, d_2, \dots, d_{20}$ respectively. Since $d_{i+1} \not\subset d_i \forall i \in \{1,2,\dots,20\}$, then we get the table (1).

Lemma (3). The block B_3 is associate and the decomposition matrix for it's a table (2).

Table (2)

⟨25,2⟩	1																																							
⟨25,2⟩′		1																																						
⟨24,3⟩	1		1																																					
⟨24,3⟩′		1		1																																				
⟨22,3,2⟩*			1	1	1	1																																		
⟨21,3,2,1⟩					1		1																																	
⟨21,3,2,1⟩′						1		1																																
⟨18,4,3,2⟩							1		1																															
⟨18,4,3,2⟩′								1		1																														
⟨17,5,3,2⟩									1		1																													
⟨17,5,3,2⟩′										1		1																												
⟨16,6,3,2⟩									1		1		1																											
⟨16,6,3,2⟩′										1		1		1																										
⟨15,7,3,2⟩							1		1				1		1																									
⟨15,7,3,2⟩′								1		1				1		1																								
⟨14,13⟩			1														1																							
⟨14,13⟩′				1														1																						
⟨14,11,2⟩*	1	1	1	1	1	1											1	1	1	1																				
⟨14,10,2,1⟩					1		1												1		1																			
⟨14,10,2,1⟩′						1		1												1		1																		
⟨14,8,3,2⟩							1								1						1		1																	
⟨14,8,3,2⟩′								1								1						1		1																
⟨14,7,4,2⟩													1		1								1		1															
⟨14,7,4,2⟩′														1		1								1		1														
⟨14,6,5,2⟩													1												1															
⟨14,6,5,2⟩′														1												1														
⟨13,11,3⟩*	1	1															1	1	1	1							1	1												
⟨13,10,3,1⟩																	1	1	1		1						1		1											
⟨13,10,3,1⟩′																	1	1		1		1						1		1										
⟨13,9,3,2⟩																					1		1						1		1									
⟨13,9,3,2⟩′																						1		1						1		1								
⟨13,7,4,3⟩																							1		1						1		1							
⟨13,7,4,3⟩′																								1		1						1		1						
⟨13,6,5,3⟩																									1								1							
⟨13,6,5,3⟩′																										1								1						
⟨12,10,3,2⟩																	1	1									1	1	1		1				1	1				
⟨12,10,3,2⟩′																	1	1									1	1		1		1			1	1				
⟨11,10,3,2,1⟩																											1	1			1	1			1	1	1	1		
⟨11,7,4,3,2⟩*																															1	1	1	1	1	1	1	1	1	1
⟨11,6,5,3,2⟩*																																	1	1					1	1
⟨10,7,4,3,2,1⟩																																			1				1	
⟨10,7,4,3,2,1⟩																																				1				1
⟨10,6,5,3,2,1⟩																																					1		1	
⟨10,6,5,3,2,1⟩																																						1		1
⟨7,6,5,4,3,2⟩																																					1			
⟨7,6,5,4,3,2⟩′																																						1		
	d_{61}	d_{62}	d_{63}	d_{64}	d_{65}	d_{66}	d_{67}	d_{68}	d_{69}	d_{70}	d_{71}	d_{72}	d_{73}	d_{74}	d_{75}	d_{76}	d_{77}	d_{78}	d_{79}	d_{80}	d_{81}	d_{82}	d_{83}	d_{84}	d_{85}	d_{86}	d_{87}	d_{88}	d_{89}	d_{90}	d_{91}	d_{92}	d_{93}	d_{94}	d_{95}	d_{96}	d_{97}	d_{98}	d_{99}	d_{100}

Proof. Using $(r,\bar{r})$-inducing of S_{26} to S_{27} we get on approximation matrix in table(3).

Table (3)

⟨25,2⟩	1																									a_1	
⟨25,2⟩′	1																										a_1
⟨24,3⟩	1	1																								a_2	a_2
⟨24,3⟩′	1	1																								a_3	
⟨22,3,2⟩*		2	1	1																							a_3
⟨21,3,2,1⟩			1		1																					a_4	
⟨21,3,2,1⟩′				1	1																						a_4
⟨18,4,3,2⟩					1	1																				a_5	
⟨18,4,3,2⟩′					1	1																					a_5
⟨17,5,3,2⟩						1	1																			a_6	
⟨17,5,3,2⟩′						1	1																				a_6
⟨16,6,3,2⟩						1	1	1																		a_7	
⟨16,6,3,2⟩′						1	1	1																			a_7
⟨15,7,3,2⟩					1	1		1	1																	a_8	
⟨15,7,3,2⟩′					1	1		1	1																		a_8
⟨14,13⟩		1								1																a_9	
⟨14,13⟩′		1								1																	a_9
⟨14,11,2⟩*	2	2	1	1						2	1	1														a_{10}	a_{10}
⟨14,10,2,1⟩			1		1						1		1													a_{11}	
⟨14,10,2,1⟩′				1	1							1	1														a_{11}
⟨14,8,3,2⟩					1				1				1	1												a_{12}	
⟨14,8,3,2⟩′					1				1				1	1													a_{12}
⟨14,7,4,2⟩								1	1					1	1											a_{13}	
⟨14,7,4,2⟩′								1	1					1	1												a_{13}
⟨14,6,5,2⟩								1							1											a_{14}	
⟨14,6,5,2⟩′								1							1												a_{14}
⟨13,11,3⟩*	2									2	1	1				1	1									a_{15}	a_{15}
⟨13,10,3,1⟩										2	1		1			1		1								a_{16}	
⟨13,10,3,1⟩′										2		1	1				1	1									a_{16}
⟨13,9,3,2⟩													1	1				1	1							a_{17}	
⟨13,9,3,2⟩′													1	1				1	1								a_{17}
⟨13,7,4,3⟩														1	1				1	1						a_{18}	
⟨13,7,4,3⟩′														1	1				1	1							a_{18}
⟨13,6,5,3⟩															1					1						a_{19}	
⟨13,6,5,3⟩′															1					1							a_{19}
⟨12,10,3,2⟩										2						1	1	1	1		1	1				a_{20}	
⟨12,10,3,2⟩′										2						1	1	1	1		1	1					a_{20}
⟨11,10,3,2,1⟩*																1	1		2		1	1	2			a_{21}	a_{21}
⟨11,7,4,3,2⟩*																			2	2	1	1	2	1	1	a_{22}	a_{22}
⟨11,6,5,3,2⟩*																				2				1	1	a_{23}	a_{23}
⟨10,7,4,3,2,1⟩																					1			1		a_{24}	
⟨10,7,4,3,2,1⟩′																						1			1		a_{24}
⟨10,6,5,3,2,1⟩																							1	1		a_{25}	
⟨10,6,5,3,2,1⟩′																							1		1		a_{25}
⟨7,6,5,4,3,2⟩																							1			a_{26}	
⟨7,6,5,4,3,2⟩′																							1				a_{26}
	k_1	k_2	d_{65}	d_{66}	k_3	k_4	k_5	k_6	k_7	k_8	d_{79}	d_{80}	k_9	k_{10}	k_{11}	d_{87}	d_{88}	k_{12}	k_{13}	k_{14}	d_{95}	d_{96}	k_{15}	d_{99}	d_{10}	Y_1	Y_2

First $(k_4 - k_5) \downarrow_{(2,10)} S_{26}$ is not p.s., so $k_5 \not\subset k_4$[3]. Since $\langle 25,2\rangle \neq \langle 25,2\rangle'$ on $(11,\alpha)$-regular classes[2] so $k_1 = d_{61} + d_{62}$ or there columns Y_1, Y_2 . If there are it so we have the approximation matrix as above, to describe it's such that $\langle 25,2\rangle \downarrow S_{26}$ has two of i.m.s, and from table(3) $a_1 \in \{0,1\}$, If $a = 1$, but $B_3$is associate[6], so k_1 must have a conjugate p.s. so $\langle 25,2\rangle$ have three m.s. contradiction since $\langle 15,3,2\rangle$ has at most two of m.s., so $a_1 = 0$ and $k_1 = d_{61} + d_{62}$. Also $\langle 24,3\rangle \downarrow S_{26}$, $\langle 21,3,2,1\rangle \downarrow S_{26}$, $\langle 18,4,3,2\rangle \downarrow S_{26}$ have 3 of i.m.s and, from table(3) we get $a_2 \in \{0,1\}, a_4 \in \{0,1\}, a_5 \in \{0,1\}$, so k_2, k_3, k_4 splits to $d_{63}, d_{64}, d_{67}, d_{68}, d_{69}, d_{70}$.

Since $\langle 17,5,3,2\rangle \neq \langle 17,5,3,2\rangle'$ so k_5 it's split or there are Y_1, Y_2. Since $\langle 17,5,3,2\rangle \downarrow S_{26}$ has 4 of i.m.s and, from table(3)we find $a_6 \in \{0,1,2\}$. Same way we find $a_9, a_{26} = 0, a_{25} \in \{0,1,2\}, a_7, a_8, a_{12}, a_{14}, a_{19}, a_{21}, a_{23}, a_{24} \in \{0,1,2,3\}, a_{10}, a_{11}, a_{15}, a_{18}, a_{22} \in \{0,1,\dots,4\}$, a_{13}, $a_{20} \in \{0,1,\dots,5\}$. Let $a_6 \in \{1,2\}$(if $a_6 = 0$ contradiction), $\langle 17,5,3,2\rangle \downarrow S_{26} \cap \langle 14,11,2\rangle \downarrow S_{25}$ has no i.m.s so we have $a_{10} = 0$ by counting the intersections we get on $a_{12}, a_{13}, \dots, a_{25}$ are equal to zero. Then deg $Y_1, Y_2 \equiv 0\ mod 11^2$ only when $Y_1 + Y_2 = k_5$, so $k_5 = d_{71}, +d_{72}$.

In $k_6$we have $\langle 16,6,3,2\rangle \neq \langle 16,6,3,2\rangle'$ so it's split or there are two columns Y_1, Y_2. Let $a_7 \in \{1,2,3\}$, when calculating intersections inducing of i.m.s we get on $a_{10}, a_{11}, a_{12}, a_{15}, a_{16}, \dots, a_{25}$ are equal to zero, then deg$Y_1, Y_2 \equiv 0\ mod 11^2$ only when $a_7 = a_8 = a_{13} = a_{14}$, so $k_6 = d_{73} + d_{74}$.

For k_7 we have $\langle 15,7,3,2\rangle \neq \langle 15,7,3,2\rangle'$suppose there are two columns Y_1, Y_2 are splits. Let $a_8 \in \{1,2,3\}$, since $\langle 15,7,3,2\rangle \downarrow S_{26} \cap \langle 14,11,2\rangle \downarrow S_{25}$ has no i.m.s so we have $a_{10} = 0$, and $\langle 15,7,3,2\rangle \downarrow S_{26} \cap \langle 14,10,2,1\rangle \downarrow S_{25}$ has one i.m.s so we have $a_{11} = 0$ [2],[3] by counting the intersections we get on $a_{15}, a_{16}, \dots, a_{25}$ are equal to zero, and since inducing m.s. is m.s. [8] we have:

$$(\langle 15,7,3,1\rangle^* - \langle 14,8,3,1\rangle^* + \langle 13,9,3,1\rangle^*) \uparrow^{(2,10)} S_{27} \Rightarrow a_8 \geq a_{12} \quad (1)$$

Then deg$Y_1, Y_2 \equiv 0\ mod\ 11^2$ only when $a_8 = a_{12} = a_{13}$ and, $a_{14} = 0$ so $k_8 = d_{75} + d_{76}$.

Since$\langle 14,13\rangle \neq \langle 14,13\rangle'$, $\langle 7,6,5,4,3,2\rangle \neq \langle 7,6,5,4,3,2\rangle'$ and $\langle 14,13\rangle \downarrow S_{26}$ has 2 of i.m.s., $\langle 7,6,5,4,3,2\rangle \downarrow S_{26}$ has 1 of i.m.s. and, from table (3) so we have $a_9 = a_{26} = 0$ ($a_9 = a_{26} = 1$ give a contradiction). Then $k_8 = d_{77} + d_{78}$, $k_{15} = d_{97} + d_{98}$

In $k_9$we have $\langle 14,8,3,2\rangle \neq \langle 14,8,3,2\rangle'$. Let $a_{12} \in \{1,2,3\}$, then its' splits or there are another two columns are splits, if used the same above we get on:

$Y_1 = a_{11}\langle 14,10,2,1\rangle + a_{12}\langle 14,8,3,2\rangle + a_{13}\langle 14,7,4,2\rangle + a_{16}\langle 13,10,3,1\rangle + a_{17}\langle 13,9,3,2\rangle + a_{18}\langle 13,7,4,3\rangle + a_{20}\langle 12,10,3,2\rangle$, $Y_2 = a_{11}\langle 14,10,2,1\rangle' + a_{12}\langle 14,8,3,2\rangle' + a_{13}\langle 14,7,4,2\rangle' + a_{16}\langle 13,10,3,1\rangle' + a_{17}\langle 13,9,3,2\rangle' + a_{18}\langle 13,7,4,3\rangle' + a_{20}\langle 12,10,3,2\rangle'$. Since inducing m.s. is m.s.[8] so we have:

$(\langle 14,6,4,2\rangle^* - \langle 13,6,4,3\rangle^* + \langle 11,6,4,3,2\rangle) \uparrow^{(5,7)} S_{27} \Rightarrow a_{13} \geq a_{18}$ (2)

$(\langle 13,6,4,3\rangle^* - \langle 14,6,4,2\rangle^* + \langle 15,6,3,2\rangle^*) \uparrow^{(5,7)} S_{27} \Rightarrow a_{18} \geq a_{13}, \therefore a_{13} = a_{18}$ (3)

$(\langle 12,9,3,2\rangle^* - \langle 12,10,3,1\rangle^* + 2\langle 14,11,1\rangle + \langle 11,9,3,2,1\rangle) \uparrow^{(2,10)} S_{27} \Rightarrow a_{17} \geq a_{16}$ (4)

$(\langle 13,10,2,1\rangle^* + \langle 13,7,4,2\rangle^* - \langle 13,8,3,2\rangle^*) \uparrow^{(3,9)} S_{27} \Rightarrow a_{11} + a_{16} + 2a_{13} \geq a_{12} + a_{17}$ (5)

$(\langle 13,8,3,2\rangle^* - \langle 13,10,2,1\rangle^* - \langle 13,7,4,2\rangle^* + \langle 13,6,5,2\rangle^* + \langle 13,11,2\rangle) \uparrow^{(3,9)} S_{27} \Rightarrow a_{12} + a_{17} \geq a_{11} + a_{16} + 2a_{13}, \therefore a_{12} + a_{17} = a_{11} + a_{16} + 2a_{13}$ (6)

$(\langle 13,10,3\rangle + \langle 13,10,3\rangle' - \langle 14,10,2\rangle - \langle 14,10,2\rangle' - \langle 11,10,3,2\rangle^* + \langle 21,3,2\rangle + \langle 21,3,2\rangle' + \langle 10,7,4,3,2\rangle + \langle 10,7,4,3,2\rangle') \uparrow^{(0,1)} S_{27} \Rightarrow 2a_{16} \geq a_{20} + 2a_{11}$ (7)

$(\langle 11,10,3,2\rangle^* + \langle 14,10,2\rangle + \langle 14,10,2\rangle' - \langle 13,10,3\rangle - \langle 13,10,3\rangle') \uparrow^{(0,1)} S_{27} \Rightarrow a_{20} + 2a_{11} \geq 2a_{16}, \therefore 2a_{16} = 2a_{11} + a_{20}$ (8)

Then deg$Y_1, Y_2 \equiv 0 \ mod \ 11^2$ only when $Y_1 + Y_2 = n_1k_9 + n_2k_{10},\ n_1 = 0, n_2 \in \{1,2,3\}$,

$or\ n_1 \in \{1,2,3\}, n_2 \in \{0,1,\dots,3-n_1\}$ so $k_9 = d_{81} + d_{82}$.

Since$\langle 14,6,5,2\rangle \neq \langle 14,6,5,2\rangle'$ and, let $a_{14} \in \{1,2,3\}$ using the same technic we get:

$Y_1 = a_{13}\langle 14,7,4,2\rangle + a_{14}\langle 14,6,5,2\rangle + a_{18}\langle 13,7,4,3\rangle + a_{19}\langle 13,6,5,3\rangle,\ Y_2 = a_{13}\langle 14,7,4,2\rangle' + a_{14}\langle 14,6,5,2\rangle' + a_{18}\langle 13,7,4,3\rangle' + a_{19}\langle 13,6,5,3\rangle'$ and since:

$(\langle 12,6,5,3\rangle^* - \langle 12,7,4,3\rangle^* + \langle 12,9,3,2\rangle^*) \uparrow^{(2,10)} S_{27} \Rightarrow a_{19} \geq a_{18}$ (9)

Then deg$Y_1, Y_2 \equiv 0 \ mod \ 11^2$ only when $Y_1 + Y_2 = k_{11}$, so $k_{11} = d_{85} + d_{86}$.

Since$\langle 14,7,4,2\rangle \neq \langle 17,4,2\rangle'$so k_{10} split or there are two columns. Let $a_{13} \in \{1,\dots,5\}$ by restricting and inducing we get: $Y_1 = a_{12}\langle 14,8,3,2\rangle + a_{13}\langle 14,7,4,2\rangle + a_{14}\langle 14,6,5,2\rangle + a_{17}\langle 13,9,3,2\rangle + a_{18}\langle 13,7,4,3\rangle + a_{19}\langle 13,6,5,3\rangle, Y_2 = a_{12}\langle 14,8,3,2\rangle' + a_{13}\langle 14,7,4,2\rangle' + a_{14}\langle 14,6,5,2\rangle' + a_{17}\langle 13,9,3,2\rangle' + a_{18}\langle 13,7,4,3\rangle' + a_{19}\langle 13,6,5,3\rangle'$,and :

$(\langle 14,6,5,1\rangle^* - \langle 12,6,5,3\rangle^* + \langle 11,7,4,3,1\rangle) \uparrow^{(2,10)} S_{27} \Rightarrow a_{14} \geq a_{19}$ (10)

$(\langle 12,6,5,3\rangle^* - \langle 14,6,5,1\rangle^* + \langle 15,7,3,1\rangle^*) \uparrow^{(2,10)} S_{27} \Rightarrow a_{19} \geq a_{14}, \therefore a_{14} = a_{19}$ (11)

and, from (3.2),(3,3)We have $\deg Y_1, Y_2 \equiv 0\ mod\ 11^2$ only when $Y_1 + Y_2 = n_1 k_{11} + n_2 k_{10}$, such that $n_1 \in \{1,2,3\}, n_2 \in \{0, \dots, 3 - n_1\}$, or $n_1 = 0, n_2 \in \{1, \dots, 3\}$ so $k_{10} = d_{83} + d_{84}$.

Since $\langle 13,10,3,1\rangle \neq \langle 13,10,31\rangle'$ then k_{12} is divided or there are Y_1, Y_2. Let $a_{16} \in \{1, \dots, 10\}$ by restricting and inducing we get:$Y_1 = a_{16}\langle 13,10,3,1\rangle + a_{17}\langle 13,9,3,2\rangle + a_{18}\langle 13,7,4,3\rangle + a_{20}\langle 12,10,3,2\rangle + a_{21}\langle 11,10,3,2,1\rangle^* + a_{22}\langle 11,7,4,3,2\rangle^* + a_{24}\langle 10,7,4,3,2,1\rangle$, $Y_2 = a_{16}\langle 13,10,3,1\rangle' + a_{17}\langle 13,9,3,2\rangle' + a_{18}\langle 13,7,4,3\rangle' + a_{20}\langle 12,10,3,2\rangle' + a_{21}\langle 11,10,3,2,1\rangle^* + a_{22}\langle 11,7,4,3,2\rangle^* + a_{24}\langle 10,7,4,3,2,1\rangle'$ and we have:

$(\langle 11,7,4,3,1\rangle - \langle 11,9,3,2,1\rangle + \langle 12,11,3\rangle) \uparrow^{(2,10)} S_{27} \Rightarrow a_{22} \geq a_{21}$ (12)

$(\langle 11,9,3,2,1\rangle - \langle 11,7,4,3,1\rangle + \langle 11,6,5,3,1\rangle) \uparrow^{(2,10)} S_{27} \Rightarrow a_{21} \geq a_{22}, \therefore a_{21} = a_{22}$ (13)

$(\langle 13,7,4,2\rangle^* + \langle 13,10,2,1\rangle^* - \langle 13,8,3,2\rangle^*) \uparrow^{(3,9)} S_{27} \Rightarrow a_{18} + a_{16} \geq a_{17}$ (14)

$(\langle 13,8,3,2\rangle^* - \langle 13,7,4,2\rangle^* - \langle 13,10,21\rangle^* + \langle 13,112\rangle + \langle 13,6,52\rangle) \uparrow^{(3,9)} S_{27} \Rightarrow a_{17} \geq a_{18} + a_{16}, \therefore a_{17} = a_{16} + a_{18}$ (15)

$(\langle 11,10,3,2\rangle^* - \langle 10,7,4,3,2\rangle + \langle 10,6,5,3,2\rangle) \uparrow^{(0,1)} S_{27} \Rightarrow a_{20} \geq a_{24}$ (16)

$(\langle 13,6,4,3\rangle^* - \langle 11,6,4,3,2\rangle + \langle 10,6,4,3,2,1\rangle^*) \uparrow^{(5,7)} S_{27} \Rightarrow a_{18} \geq a_{21}$ (17)

$(\langle 11,6,4,3,2,1\rangle^* - \langle 13,6,4,3\rangle^* + \langle 14,6,4,2\rangle^*) \uparrow^{(5,7)} S_{27} \Rightarrow a_{21} \geq a_{18}, \therefore a_{18} = a_{21}$ (18)

Then $\deg Y_1, Y_2 \equiv 0\ mod\ 11^2$ only when $Y_1 + Y_2 = n_1 k_{12} + n_2 k_{13}$, $n_1 = 1, n_2 \in \{0,1, \dots, 3\}$ or $n_1 \in \{2,3, \dots, 5\}$, $n_2 \in \{0,1, \dots, 5 - n_1\}$ so $k_{12} = d_{89} + d_{90}$.

For k_{13} $\langle 13,9,3,2\rangle \neq \langle 13,9,3,2\rangle'$, let $a_{17} \in \{1,2, \dots, 8\}$ by intersection restricting, and since:

$(\langle 13,6,4,3\rangle^* - \langle 11,6,4,3,2\rangle + \langle 10,6,4,3,2,1\rangle) \uparrow^{(5,7)} S_{27} \Rightarrow a_{18} \geq a_{22}$ (19)

$(\langle 11,6,4,3,2\rangle - \langle 13,6,4,3\rangle^* + \langle 14,6,4,2\rangle^*) \uparrow^{(5,7)} S_{27} \Rightarrow a_{22} \geq a_{18}, \therefore a_{18} = a_{22}$ (20)

$(\langle 13,8,3,2\rangle^* - \langle 13,7,4,2\rangle^* + \langle 13,6,5,2\rangle^*) \uparrow^{(3,9)} S_{27} \Rightarrow a_{17} \geq a_{18}$ (21)

$(\langle 13,7,4,2\rangle^* - \langle 13,8,3,2\rangle^* + \langle 13,10,2,1\rangle^*) \uparrow^{(3,9)} S_{27} \Rightarrow a_{18} \geq a_{17}, \therefore a_{17} = a_{18}$ (22)

And, from (12), (13) we get on $\deg Y_1, Y_2 \equiv 0\ mod\ 11^2$ only when $Y_1 + Y_2 = k_{13}$ so $k_{13} = d_{91} + d_{92}$.

For k_{14} we have $\langle 13,7,4,3\rangle \neq \langle 13,7,4,3\rangle'$, let $a_{18} \in \{1,2,\dots,4\}$ by intersection restricting, and by inducing:

$$(\langle 13,7,4,2\rangle^* - \langle 13,6,5,2\rangle^*) \uparrow^{(3,9)} S_{27} \Rightarrow a_{18} \geq a_{19} \quad (23)$$

$$(\langle 13,6,5,2\rangle^* - \langle 13,7,4,2\rangle^* + \langle 13,8,3,2\rangle^*) \uparrow^{(3,9)} S_{27} \Rightarrow a_{19} \geq a_{18}, \therefore a_{18} = a_{19} \quad (24)$$

$$(\langle 12,6,5,3\rangle^* - \langle 11,6,5,3,1\rangle + \langle 9,7,4,3,2,1\rangle^*) \uparrow^{(2,10)} S_{27} \Rightarrow a_{18} \geq a_{23} \quad (25)$$

$$(\langle 11,6,5,3,1\rangle - \langle 12,6,5,3\rangle^* + \langle 14,6,5,1\rangle^*) \uparrow^{(2,10)} S_{27} \Rightarrow a_{23} \geq a_{18}, \therefore a_{18} = a_{23} \quad (26)$$

$$(\langle 12,9,3,2\rangle^* - \langle 11,9,3,2,1\rangle + \langle 9,6,5,3,2,1\rangle^*) \uparrow^{(2,10)} S_{27} \Rightarrow a_{20} \geq a_{21} \quad (27)$$

$$(\langle 11,9,3,2,1\rangle - \langle 12,9,3,2\rangle + \langle 9,7,4,3,2,1\rangle^* + \langle 13,9,3,1\rangle^*) \uparrow^{(2,10)} S_{27} \Rightarrow a_{21} \geq a_{20}, \therefore a_{20} = a_{21} \quad (28)$$

we get on deg$Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $Y_1 + Y_2 = k_{14}$ so $k_{14} = d_{93} + d_{94}$. Therefore table(3) is decomposition matrix for the block B_3 .

Theorem (4). Decomposition matrix for the block B_1 is table (4).

Table (4)

$\langle 26,1\rangle$	1																																							
$\langle 26,1\rangle'$		1																																						
$\langle 23,4\rangle$	1		1																																					
$\langle 23,4\rangle'$		1		1																																				
$\langle 22,4,1\rangle^*$			1	1	1	1																																		
$\langle 20,4,2,1\rangle$					1		1																																	
$\langle 20,4,2,1\rangle'$						1		1																																
$\langle 19,4,3,1\rangle$							1		1																															
$\langle 19,4,3,1\rangle'$								1		1																														
$\langle 17,5,4,1\rangle$									1		1																													
$\langle 17,5,4,1\rangle'$										1		1																												
$\langle 16,6,4,1\rangle$									1		1		1																											
$\langle 16,6,4,1\rangle'$										1		1		1																										
$\langle 15,12\rangle$			1												1																									
$\langle 15,12\rangle'$				1												1																								
$\langle 15,11,1\rangle^*$	1	1	1	1	1	1									1	1	1	1																						
$\langle 15,9,2,1\rangle$					1		1										1		1																					
$\langle 15,9,2,1\rangle'$						1		1										1		1																				
$\langle 15,8,3,1\rangle$							1		1										1		1																			
$\langle 15,8,3,1\rangle'$								1		1										1		1																		
$\langle 15,7,4,1\rangle$									1				1								1		1																	
$\langle 15,7,4,1\rangle'$										1				1								1		1																
$\langle 15,6,5,1\rangle$													1										1																	
$\langle 15,6,5,1\rangle'$														1										1																
$\langle 14,8,4,1\rangle$																			1		1		1		1															

⟨14,8,4,1⟩′																				1		1		1		1														
⟨13,9,4,1⟩																	1		1						1		1													
⟨13,9,4,1⟩′																		1		1						1		1												
⟨12,11,4⟩*	1	1													1	1	1	1											1	1										
⟨12,10,4,1⟩															1	1	1										1		2		1	1								
⟨12,10,4,1⟩′															1	1		1										1		2	1	1								
⟨12,9,4,2⟩																									1		1		1		1	1	1							
⟨12,9,4,2⟩′																										1		1		1	1	1		1						
⟨12,8,4,3⟩																							1		1								1		1					
⟨12,8,4,3⟩′																								1		1								1		1				
⟨12,6,5,4⟩																							1												1					
⟨12,6,5,4⟩′																								1												1				
⟨11,9,4,2,1⟩*																													1	1	1	1	1	1			1	1		
⟨11,8,4,3,1⟩*																															1	1	1	1	1	1	1	1	1	1
⟨11,6,5,4,1⟩*																																			1	1			1	1
⟨9,8,4,3,2,1⟩																															1								1	
⟨9,8,4,3,2,1⟩																																1								1
⟨9,6,5,4,2,1⟩																																					1		1	
⟨9,6,5,4,2,1⟩																																						1		1
⟨8,6,5,4,3,1⟩																																					1			
⟨8,6,5,4,3,1⟩																																						1		
	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}	d_{37}	d_{38}	d_{39}	d_{40}	d_{41}	d_{42}	d_{43}	d_{44}	d_{45}	d_{46}	d_{47}	d_{48}	d_{49}	d_{50}	d_{51}	d_{52}	d_{53}	d_{54}	d_{55}	d_{56}	d_{57}	d_{58}	d_{59}	d_{60}

Proof. By using $(\mathrm{r},\bar{r})$-inducing of S_{26} to S_{27} we get on approximation matrix in table (5).

Table (5)

⟨26,1⟩	1																																
⟨26,1⟩′	1																																
⟨23,4⟩	1	1																														a_1	
⟨23,4⟩′	1	1																															a_1
⟨22,4,1⟩*		2	1	1																												a_2	a_2
⟨20,4,2,1⟩			1		1																											a_3	
⟨20,4,2,1⟩′				1		1																											a_3
⟨19,4,3,1⟩					1		1																									a_4	
⟨19,4,3,1⟩′						1		1																									a_4
⟨17,5,4,1⟩							1		1																							a_5	
⟨17,5,4,1⟩′								1		1																							a_5
⟨16,6,4,1⟩							1		1		1																					a_6	
⟨16,6,4,1⟩′								1		1		1																					a_6
⟨15,12⟩		1											1																			a_7	
⟨15,12⟩′		1											1																				a_7
⟨15,11,1⟩*	2	2	1	1									2	1	1																	a_8	a_8
⟨15,9,2,1⟩			1		1									1		1																a_9	
⟨15,9,2,1⟩′				1		1									1		1																a_9
⟨15,8,3,1⟩					1		1									1		1														a_{10}	
⟨15,8,3,1⟩′						1		1									1		1														a_{10}
⟨15,7,4,1⟩							1				1							1		1												a_{11}	
⟨15,7,4,1⟩′								1				1							1	1													a_{11}
⟨15,6,5,1⟩											1									1												a_{12}	

$\langle 15,6,5,1\rangle'$												1								1													a_{12}
$\langle 14,8,4,1\rangle$																1		1		1	1											a_{13}	
$\langle 14,8,4,1\rangle'$																	1		1	1	1												a_{13}
$\langle 13,9,4,1\rangle$														1		1					1	1										a_{14}	
$\langle 13,9,4,1\rangle'$															1		1				1	1											a_{14}
$\langle 12,11,4\rangle^*$	2												2	1	1								2									a_{15}	a_{15}
$\langle 12,10,4,1\rangle$													2	1								1	2	1	1							a_{16}	
$\langle 12,10,4,1\rangle$													2		1							1	2	1	1								a_{16}
$\langle 12,9,4,2\rangle$																					1	1	1	1	1	1						a_{17}	
$\langle 12,9,4,2\rangle'$																					1	1	1	1	1	1							a_{17}
$\langle 12,8,4,3\rangle$																				1	1					1	1					a_{18}	
$\langle 12,8,4,3\rangle'$																				1	1					1	1						a_{18}
$\langle 12,6,5,4\rangle$																				1							1					a_{19}	
$\langle 12,6,5,4\rangle'$																				1							1						a_{19}
$\langle 11,9,4,2,1\rangle$																							2	1	1	2		1	1			a_{20}	a_{20}
$\langle 11,8,4,3,1\rangle$																								1	1	2	2	1	1	1	1	a_{21}	a_{21}
$\langle 11,6,5,4,1\rangle$																											2			1	1	a_{22}	a_{22}
$\langle 9,8,4,3,2,1$																								1						1		a_{23}	
$\langle 9,8,4,3,2,1$																									1						1		a_{23}
$\langle 9,6,5,4,2,1$																												1		1		a_{24}	
$\langle 9,6,5,4,2,1$																													1		1		a_{24}
$\langle 8,6,5,4,3,1$																												1				a_{25}	
$\langle 8,6,5,4,3,1$																													1				a_{25}
	k_1	k_2	d_{25}	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}	d_{31}	d_{32}	d_{33}	d_{34}	k_3	d_{37}	d_{38}	d_{39}	d_{40}	d_{41}	d_{42}	k_4	k_5	k_6	k_7	d_{51}	d_{52}	k_8	k_9	d_{57}	d_{58}	d_{59}	d_{60}	Y_1	Y_2

Since $\langle 23,4\rangle \neq \langle 23,4\rangle'$[4] so $k_2 = d_{23} + d_{24}$ or there are two columns, if there are it so we have the approximation matrix as above. Since $\langle 23,4\rangle \downarrow S_{26}$ has six of i.m.s and, from table(5) we have $a_1 \in \{0,1,\dots,4\}$. In the same way find the others. Take $a_1 \in \{1,\dots,4\}$ and, $\langle 23,4\rangle \downarrow S_{26} \cap \langle 20,4,2,1\rangle \downarrow S_{26}$ has no i.m.s so we have $a_3 = 0$ by counting the intersections we get on $a_4, a_5, a_6, a_9, a_{10}, \dots, a_{14}, a_{17}, a_{18} \dots, a_{25}$ are equal to zero, and since inducing m.s. is m.s. [8] we have:

$(\langle 14,12\rangle^* - \langle 14,11,1\rangle + \langle 25,1\rangle^* + \langle 14,9,2,1\rangle^*) \uparrow^{(4,8)} S_{27} \Rightarrow a_7 \geq a_8$ (29)

$(\langle 14,11,1\rangle - \langle 14,12\rangle^*) \uparrow^{(4,8)} S_{27} \Rightarrow a_8 \geq a_7, \therefore a_7 = a_8$ (30)

$(\langle 11,10,4,1\rangle^* - \langle 12,10,4\rangle + \langle 13,9,4\rangle + \langle 26\rangle + \langle 26\rangle') \uparrow^{(0,1)} S_{27} \Rightarrow \therefore a_{15} = 0$ (31)

$(\langle 23,3\rangle^* - \langle 22,3,1\rangle + \langle 20,3,2,1\rangle^*) \uparrow^{(4,8)} S_{27} \Rightarrow a_1 \geq a_2$ (32)

$(\langle 22,3,1\rangle - \langle 23,3\rangle^* + \langle 25,1\rangle^*) \uparrow^{(4,8)} S_{27} \Rightarrow a_2 \geq a_1, \therefore a_1 = a_2$ (33)

So $\deg Y_1, Y_2 \equiv 0 \ mod\ 11^2$ only when $Y_1 + Y_2 = k_2$. By[1] we divided $k_1 = d_{21} + d_{22}$.

$\langle 15,12\rangle \neq \langle 15,12\rangle'$so k_3 splits or there are Y_1, Y_2. Use the same above, and:

$(\langle 14,12\rangle - \langle 14,11,1\rangle + \langle 25,1\rangle^* + \langle 14,9,2,1\rangle) \uparrow^{(4,8)} S_{27} \Rightarrow a_7 \geq a_8$ (34)

$(\langle 14,11,1\rangle - \langle 14,12\rangle) \uparrow^{(4,8)} S_{27} \Rightarrow a_8 \geq a_7, \therefore a_7 = a_8$ (35)

Then $\deg Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $Y_1 + Y_2 = k_3$, so $k_3 = d_{35} + d_{36}$

For k_4, $\langle 15,6,5,1\rangle \neq \langle 15,6,5,1\rangle'$ on $(11, \alpha)$ −regular classes so $k_4 = d_{43} + d_{44}$. Use the same above and, since:

$(\langle 11,6,5,3,1\rangle + \langle 11,9,3,2,1\rangle - \langle 11,7,4,3,1\rangle) \uparrow^{(4,8)} S_{27} \Rightarrow a_{22} \geq a_{21}$ (36)

$(\langle 11,7,4,3,1\rangle - \langle 11,6,5,3,1\rangle) \uparrow^{(4,8)} S_{27} \Rightarrow a_{21} \geq a_{22}, \therefore a_{21} = a_{22}$ (37)

$(\langle 15,6,5\rangle - \langle 15,7,4\rangle + \langle 15,8,3\rangle) \uparrow^{(0,1)} S_{27} \Rightarrow a_{12} \geq a_{11}$ (38)

$(\langle 15,7,4\rangle - \langle 15,6,5\rangle) \uparrow^{(0,1)} S_{27} \Rightarrow a_{11} \geq a_{12}, \therefore a_{11} = a_{12}$ (39)

$(\langle 11,6,5,4\rangle^* - \langle 11,8,4,3\rangle^* + \langle 11,9,4,2\rangle^*) \uparrow^{(0,1)} S_{27} \Rightarrow a_{19} \geq a_{18}$ (40)

$(\langle 11,8,4,3\rangle^* - \langle 11,6,5,4\rangle^*) \uparrow^{(0,1)} S_{27} \Rightarrow a_{18} \geq a_{19}, \therefore a_{18} = a_{19}$ (41)

So $\deg Y_1, Y_2 \equiv 0 mod\; 11^2$ only when $Y_1 + Y_2 = n_1 k_4 + n_2 k_9$, $n_1 \in \{0,1,2,3\}, n_2 \in \{0,1, \dots, 4 - n_1\}$ so it's splits.

In $k_6$we have $\langle 13,9,4,1\rangle \neq \langle 13,9,4,1\rangle'$so $k_6 = d_{47} + d_{48}$ or there are Y_1, Y_2 . Use the same above and, since:

$(\langle 11,10,4,1\rangle^* - \langle 12,10,4\rangle + \langle 15,9,2\rangle) \uparrow^{(0,1)} S_{27} \Rightarrow 0 > a_{15}, \therefore a_{15} = 0$ (42)

$(\langle 11,7,4,3,1\rangle - \langle 11,9,3,2,1\rangle + \langle 12,11,3\rangle) \uparrow^{(4,8)} S_{27} \Rightarrow a_{21} \geq a_{20}$ (43)

$(\langle 11,9,3,2,1\rangle - \langle 11,7,4,3,1\rangle + \langle 11,6,5,3,1\rangle) \uparrow^{(4,8)} S_{27} \Rightarrow a_{20} \geq a_{21}, \therefore a_{20} = a_{21}$ (44)

$(\langle 13,9,3,1\rangle^* - \langle 12,10,3,1\rangle^* + \langle 12,11,3\rangle + \langle 12,11,3\rangle' + 2\langle 9,7,4,3,2,1\rangle^*) \uparrow^{(4,8)} S_{27} \Rightarrow a_{14} \geq a_{16}$ (45)

$(\langle 12,10,3,1\rangle^* - \langle 13,9,3,1\rangle^* + \langle 14,8,3,1\rangle^*) \uparrow^{(4,8)} S_{27} \Rightarrow a_{16} \geq a_{14}, \therefore a_{14} = a_{16}$ (46)

$(\langle 11,7,4,3,1\rangle - \langle 12,7,4,3\rangle + \langle 14,7,4,1\rangle) \uparrow^{(4,8)} S_{27} \Rightarrow a_{20} \geq a_{18},$ (47)

$(\langle 12,7,4,3\rangle + \langle 9,7,4,3,2,1\rangle^* + \langle 7,6,5,4,3,1\rangle^* - \langle 11,7,4,3,1\rangle) \uparrow^{(4,8)} S_{27} \Rightarrow a_{18} \geq a_{20}, \therefore a_{18} = a_{19}$ (48)

Then $\deg Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $Y_1 + Y_2 = n_1 k_6 + n_2 k_8$, $n_1 \in \{1,2,\dots,7\}, n_2 \in \{0,1,\dots,6\}$, so it's splits.

For k_7 we have $\langle 12,10,4,1\rangle \neq \langle 12,10,4,1\rangle'$ so k_7 splits or there are Y_1, Y_2 . By inducing:

$(\langle 9,7,4,3,21\rangle^* + \langle 12,7,4,3\rangle^* + \langle 11,6,5,31\rangle + \langle 9,6,5,3,21\rangle - \langle 11,7,4,3,1\rangle) \uparrow^{(4,8)} S_{27} \Rightarrow a_{23} \geq a_{21}$ (49)

$(\langle 11,7,4,3,1\rangle - \langle 9,7,4,3,2,1\rangle^*) \uparrow^{(4,8)} S_{27} \Rightarrow a_{21} \geq a_{23}, \therefore a_{21} = a_{23}$ (50)

$(\langle 11,10,4,1\rangle^* - \langle 12,10,4\rangle + \langle 13,9,4\rangle + \langle 26\rangle + \langle 26\rangle') \uparrow^{(0,1)} S_{27} \Rightarrow a_{15} = 0$ (51)

$(\langle 12,9,3,2\rangle^* - \langle 12,10,3,1\rangle^* + \langle 12,11,3\rangle + \langle 12,11,3\rangle') \uparrow^{(4,8)} S_{27} \Rightarrow a_{17} \geq a_{16}$ (52)

$(\langle 12,10,3,1\rangle^* - \langle 12,9,3,2\rangle + \langle 12,7,4,3\rangle^*) \uparrow^{(4,8)} S_{27} \Rightarrow a_{16} \geq a_{17}, \therefore a_{16} = a_{17}$ (53)

$(\langle 12,8,4,2\rangle - \langle 11,8,4,2,1\rangle + \langle 8,6,5,4,2,1\rangle^*) \uparrow^{(3,9)} S_{27} \Rightarrow a_{16} \geq a_{20} + a_{21}$ (54)

$(\langle 11,8,4,2,1\rangle - \langle 12,8,4,2\rangle + \langle 13,8,4,1\rangle) \uparrow^{(3,9)} S_{27} \Rightarrow a_{20} + a_{21} \geq a_{16}, \therefore a_{16} = a_{20} + a_{21}$ (55)

$(\langle 11,9,4,2\rangle^* - \langle 12,10,4\rangle^* - \langle 10,8,4,3,1\rangle - \langle 10,8,4,3,1\rangle' + \langle 10,6,5,4,1\rangle + \langle 10,6,5,4,1\rangle' + \langle 8,6,5,4,3\rangle + \langle 8,6,5,4,3\rangle' + \langle 15,10,1\rangle + \langle 15,10,1\rangle') \uparrow^{(0,1)} S_{27} \Rightarrow a_{17} + a_{20} \geq a_{16} + 2a_{21}$ (56)

$(\langle 12,10,4\rangle^* + \langle 10,8,4,3,1\rangle + \langle 10,8,4,3,1\rangle' - \langle 11,9,4,2\rangle^* + \langle 13,9,4\rangle + \langle 13,9,4\rangle') \uparrow^{(0,1)} S_{27} \Rightarrow a_{16} + 2a_{21} \geq a_{17} + a_{20}, \therefore a_{17} + a_{20} = a_{16} + 2a_{21}$ (57)

We, get Y_1, Y_2 which is not p.s. since $\deg Y_1, Y_2 \not\equiv 0 mod 11^2$, so $k_7 = d_{49} + d_{50}$.

For k_8 we have $\langle 12,9,4,2\rangle \neq \langle 12,9,4,2\rangle'$, so k_8 splits or there are Y_1, Y_2 . By inducing:

$(\langle 12,7,4,3\rangle^* - \langle 12,9,3,2\rangle^* + \langle 12,10,3,1\rangle^*) \uparrow^{(4,8)} S_{27} \Rightarrow a_{18} \geq a_{17}$ (58)

$(\langle 12,9,3,2\rangle^* - \langle 12,7,4,3\rangle^* + \langle 12,6,5,3\rangle^*) \uparrow^{(4,8)} S_{27} \Rightarrow a_{17} \geq a_{18}, \therefore a_{17} = a_{18}$ (59)

$(\langle 11,9,3,2,1\rangle - \langle 12,7,4,3\rangle^* + \langle 12,6,5,3\rangle^* + \langle 13,9,3,1\rangle^*) \uparrow^{(4,8)} S_{27} \Rightarrow a_{20} \geq a_{17}$ (60)

$(\langle 12,7,4,3\rangle^{*} - \langle 11,9,3,2,1\rangle + \langle 9,6,5,3,2,1\rangle^{*} + \langle 12,10,3,1\rangle^{*}) \uparrow^{(4,8)} S_{27} \Rightarrow$
$a_{17} \geq a_{20}, \therefore a_{17} = a_{20}$
(61)

Then $\deg Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $Y_1 + Y_2 = k_8$, so $k_8 = d_{53} + d_{54}$.

In k_9, $\langle 12,8,4,3\rangle \neq \langle 12,8,4,3\rangle'$ if there are two columns by restricting, inducing we get on:

$(\langle 12,6,5,3\rangle^{*} + \langle 12,9,3,2\rangle^{*} - \langle 12,7,4,3\rangle^{*}) \uparrow^{(4,8)} S_{26} \Rightarrow a_{19} \geq a_{18}$
(62)
$(\langle 12,7,4,3\rangle^{*} - \langle 12,6,5,3\rangle^{*}) \uparrow^{(4,8)} S_{26} \Rightarrow a_{18} \geq a_{19}, \therefore a_{18} = a_{19}$
(63)
$(\langle 11,8,4,2,1\rangle - \langle 12,8,4,2\rangle^{*} + \langle 13,8,4,1\rangle^{*}) \uparrow^{(3,9)} S_{26} \Rightarrow a_{20} + a_{21} \geq a_{18}$
(64)
$(\langle 12,8,4,2\rangle^{*} - \langle 11,8,4,2,1\rangle + \langle 8,6,5,4,2,1\rangle^{*}) \uparrow^{(3,9)} S_{26} \Rightarrow a_{18} \geq a_{20} + a_{21}, \therefore$
$a_{18} = a_{20} + a_{21}$
(65)

Then $\deg Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $Y_1 + Y_2 = k_8$, so $k_9 = d_{55} + d_{56}$. Finally we have 241 columns, then $k_5 = d_{45} + d_{46}$ Therefore, the decomposition matrix for the block B_2 is table (5).

Conflict of Interests.
There are non-conflicts of interest

III. References

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المشخصات المعيارية قياس $p = 11$ لزمرة التمثيل $\bar{S}_{27}$,

أحمد حسين جاسم

قسم الرياضيات, كلية العلوم, جامعة البصرة, البصرة, العراق

ahmedhussein1981@ymail.com

الخلاصة

في هذا البحث وجدنا مصفوفة التجزئة للزمرة $\bar{S}_{27}$ قياس 11 باستخدام طريقة $(r,\bar{r})$–inducing