Semi-Normed Difference Operator Triple Sequence Spaces Defined by a Double Orlicz-Functions

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Abstract

In this paper we introduce semi-normed difference operator triple sequence spaces by using a double Orlicz-functions, so we study their different properties like completeness, solidity, monotonicity, symmetricity etc.

Key words. Double Orlicz-functions, difference spaces , Triple sequences.

Introduction

Throughout this paper Ω^3 a symbol for the family of all complex or real triple sequences. Atriple sequences (complex or real) in [2][4] be a function from $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ (natural numbers) to (real or complex numbers$\mathbb{R}(\mathbb{C})$), and we denote $3c_0$ all null spaces, $3c$ the spaces of convergent, $3l_\infty$be a linear spaces

Some new results of triple sequences spaces would studied by a double Orlicz functions using a function F where $F = (F_1(r), F_2(u))$,let $(x, y) = (x_{r,u,j}, y_{r,u,j})$ be a triple infinite array of elements.

Kizmaz [6] introduced single difference of single sequence space. We use this idea to define difference of triple sequence spaces as following:

$$\Omega(\Delta) = \{ (x_r)) \in \Omega : (\Delta x_r) \in \Omega\},$$

for $\Omega = c\ , c_0, l_\infty$, where $\Delta x_r = x_r - x_{r+1}$ for all $r \in \mathbb{N}$.

The differences double sequence $(\Delta x, \Delta y)$ is defined by:

$$(\Delta x, \Delta y) = (\Delta x_{r,u}, \Delta y_{r,u})_{r,u=1}^{\infty}.$$

Let $\Omega^3(\Delta_{r,u,j})$the difference triple sequence spaces defined by :

$\Delta_{r,u,j}(x_{r,u,j}) = x_{r,u,j} - x_{r,u+1,j} - x_{r,u,j+1} + x_{r,u+1,j+1} - x_{1+r,u,1+j} - x_{1+r,1+u,j} + x_{1+r,u,1+j} - x_{1+r,u,1+j} + x_{1+r,1+u,1+j}$

In this paper we introduced the vector-valued triple sequence spaces by a double Orlicz-functions F where $F(x, y) = (F_1(x), F_2(y))$ over semi-normed space (X^2, q) semi-normed by q, and construct some important properties of triple sequence spaces by a function F.

Maysoon [7] and Zainab [3] defined the N-function as follows:

Definition .1.1[3]. The double Orlicz functions is afunction

$F: [0,\infty) \times [0,\infty) \to: [0,\infty) \times [0,\infty)\ such\ that\ F(u, v) = (F_1(u), F_2(v))$

$F_1: [0,\infty) \to [0,\infty)\ and: F_2\ [0,\infty) \to [0,\infty)$,

such that F_1,F_2 are Orlicz functions which are ,convex, non−decreasing ,even , continuous, and satisfies the four conditions :

$i)\ F_1(0) = 0\ , F_2(0) = 0 \to F(0,0) = (F_1(0)\ , F_2(0))$

$ii)\ F_1(u) > 0\ , F_2(v) > 0 \to F(u, v) = (F_1(u)\ , (F_2(v)) > (0,0)\ for\ u > 0$,

$v > 0\ , we\ mean\ that\ by F(u, v) > (0,0)\ that\ F_1(u) > 0\ , F_2(v) > 0$

$iii)\ F_1(u) \to \infty\ , F_2(v) \to \infty\ as\ u\ , v \to \infty\ then$

$F(u, v) = \big(F_1(u), F_2(v)\big) \to (\infty, \infty)\ as\ (u, v) \to (\infty, \infty) we\ mean\ by$

$F(u, v) \to (\infty, \infty), that F_1(u) \to \infty, F_2(v) \to \infty.$

Definition.2.1. Atriple sequence spaces Ω^3 is called solidary if $(\alpha_{r,u,j}x_{r,u,j}, \beta_{r,u,j}y_{r,u,j}) \in \Omega^3$ whenever $(x_{r,u,j}, y_{r,u,j}) \in \Omega^3$ for all triple sequence $(\alpha_{r,u,j}, \beta_{r,u,j})$ of scalars with $|\alpha_{r,u,j}| \leq 1$,$|\beta_{r,u,j}| \leq 1$ and consequently $|(\alpha_{r,u,j}, \beta_{r,u,j})| \leq 1$, for all r, u and $j \in \mathbb{N}$.

Definition.3.1. Atriple sequence space Ω^3 is called monotone if it consists of the canonical- pre−images of all its step space.

Definition.4.1. Atriple sequence spaces Ω^3 is called symmetric if $(x_{r,u,j}, y_{r,u,j}) \in \Omega^3$ implies $(x_{\pi(r),\pi(u),\pi(j)}, y_{\pi(r),\pi(u),\pi(j)}) \in \Omega^3$,where π is a permutation of $\mathbb{N}$.

Definition5.1. Let F be a double Orlicz-functions , $\varphi \geq o$ is real numbers and $p = (p_{r,u,j})$ be a factorable triple sequence of positive real .

Definition6.1. Let F be a double Orlicz functions which satisfies Δ_2 −condition and let $0 < \delta < 1$, then for each $x \geq \delta, y \geq \delta$ we have $F(x, y) < k(x, y)\frac{1}{\delta}F(2)$ for some constant $k > 0$.

Now we define difference operator triple sequence spaces $3l_\infty, 3c, 3c_0$ on asemi-normed space(X^2, q) seminormed by q by F as following:

$$3c(\Delta_n^v, F, p, \varphi, q) = \left\{ (x, y) \in \Omega^3 : p - lim_{r,u,j\to\infty} (ruj)^{-\varphi} \left[\left(F_1\left(\frac{q|\Delta_n^v x_{r,u,\square} - l_1|}{\rho}\right)\right) \square \left(F_2\left(\frac{q|\Delta_u^v y_{r,u,\square} - l_2|}{\rho}\right)\right) \right]^{p_{r,u,j}} = 0 , for\ some\ \square_1, \square_2 \in \mathbb{C}, \varphi \geq 0, \rho > 0 \right\}$$

,where $\left(F_1\left(\frac{q|\Delta_n^v x_{r,u,\square} - l_1|}{\rho}\right)\right) = 0$ and $\left(F_2\left(\frac{q|\Delta_u^v y_{r,u,\square} - l_2|}{\rho}\right)\right) = 0$.

$$3c_0(\Delta_n^v, F, p, \varphi, q) = \left\{ (x, y) \in \Omega^3 : p - lim_{r,u,j\to\infty} (ruj)^{-\varphi} \left[\left(F_1\left(\frac{q|\Delta_n^v x_{r,u,\square}|}{\rho}\right)\right) \square \left(F_2\left(\frac{q|\Delta_n^v y_{r,u,\square}|}{\rho}\right)\right) \right]^{p_{r,u,j}} = 0\ for\ some\ \rho > 0, \varphi \geq 0 \right\},$$

where $\left(F_1\left(\frac{q|\Delta_n^v x_{r,u,\square}|}{\rho}\right)\right) = 0$, and $\left(F_2\left(\frac{q|\Delta_n^v y_{r,u,\square}|}{\rho}\right)\right) = 0$.

And :

$$3\square_\infty(\Delta_n^v, F, p, \varphi, q) = \left\{ (x, y) \in \Omega^3 : sup_{r,u,\square} (ruj)^{-\varphi} \left[\left(F_1\left(\frac{q|\Delta_n^v x_{r,u,\square}|}{\rho}\right)\right) \square \left(F_2\left(\frac{q|\Delta_n^v y_{r,u,\square}|}{\rho}\right)\right) \right]^{p_{r,u,j}} < \infty, for\ some\ \rho > 0, l_1, l_2 \in \mathbb{C}\ and\ \varphi \geq 0 \right\}$$

,where $\left(F_1\left(\frac{q|\Delta_n^v x_{r,u,\square}|}{\rho}\right)\right) < \infty$, and $\left(F_2\left(\frac{q|\Delta_n^v y_{r,u,\square}|}{\rho}\right)\right) < \infty$.

By meaning of Mursuleen[5],we can defined difference operator triple sequence spaces $3l_{\infty}, 3c, 3c_0$ on a semi−normed (X^2, q) semi-normed by q as follows:

$$3\square_{\infty}(\Delta, F, q) = \left\{ (x, y) \in \Omega^3 : sup_{r,u,\square} \left[\left(F_1\left(\frac{q|\Delta x_{r,u,\square}|}{\rho}\right)\right) \square \left(F_2 \left(\frac{q|\Delta y_{r,u,\square}|}{\rho}\right)\right) \right] < \infty, for\ some\ \rho > 0 \right\}$$, where $\left(F_1\left(\frac{q|\Delta x_{r,u,\square}|}{\rho}\right)\right) < \infty$ and $\left(F_2 \left(\frac{q|\Delta y_{r,u,\square}|}{\rho}\right)\right) < \infty$.

$$3c_0(\Delta, F, q) = \left\{ (x, y) \in \Omega^3 : p - \lim_{r,u,j\to\infty} \left[\left(F_1\left(\frac{q|\Delta x_{r,u,\square}|}{\rho}\right)\right) \square \left(F_2 \left(\frac{q|\Delta y_{r,u,\square}|}{\rho}\right)\right) \right] = 0, for\ some\ \rho > 0 \right\}$$, where $\left(F_1\left(\frac{q|\Delta x_{r,u,\square}|}{\rho}\right)\right) = 0$, and $\left(F_2 \left(\frac{q|\Delta y_{r,u,\square}|}{\rho}\right)\right) = 0$.

And:

$$3c\,(\Delta, F, q) = \left\{ (x, y) \in \Omega^3 : p - \lim_{r,u,j\to\infty} \left[\left(F_1\left(\frac{q|\Delta x_{r,u,\square} - l_1|}{\rho}\right)\right) \square \left(F_2 \left(\frac{q|\Delta y_{r,u,\square} - l_2|}{\rho}\right)\right) \right] = 0, for\ some\ l_1, l_2 \in \mathbb{C}, \rho > 0 \right\}$$, where $\left(F_1\left(\frac{q|\Delta x_{r,u,\square} - l_1|}{\rho}\right)\right) = 0$ and $\left(F_2 \left(\frac{q|\Delta y_{r,u,\square} - l_2|}{\rho}\right)\right) = 0$

also ,by Asma [1] we can defined difference operator triple sequence spaces $3l_{\infty}, 3c, 3c_0$ on asemi-normed (X^2, q)semi−normed by q, as follows :

$$3\square_{\infty}(\square, \Delta, F, q) = \left\{ (x, y) \in \Omega^3 : sup_{r,u,\square} \left[\left(F_1\left(\frac{q|\square_{r,u,\square}\Delta x_{r,u,\square}|}{\rho}\right)\right) \square \left(F_2\left(\frac{q|\square_{r,u,\square}\Delta y_{r,u,\square}|}{\rho}\right)\right) \right] < \infty, for\ some\ \rho > 0 \right\}.$$

$$3c\,(\square, \Delta, F, q) = \left\{ (x, y) \in \Omega^3 : p - \lim_{r,u,j\to\infty} \left[\left(F_1\left(\frac{q|\square_{r,u,\square}\Delta x_{r,u,\square} - l_1|}{\rho}\right)\right) \square \left(F_2\left(\frac{q|\square_{r,u,\square}\Delta y_{r,u,\square} - l_2|}{\rho}\right)\right) \right] = 0, for\ some\ l_1, l_2 \in \mathbb{C}, \rho > 0 \right\}$$

$3c_0 \quad (□ \quad, \Delta, F, q) = \Big\{ (x, y) \in \Omega^3 : p - \lim_{r,u,j\to\infty} \Big[\Big(F_1(\frac{q|□_{r,u},□\Delta x_{r,u},□|}{\rho}) \Big) \quad □ \quad \Big(F_2 \quad (\frac{q|□_{r,u},□\Delta y_{r,u},□|}{\rho}) \Big) \Big] = 0 \; for \; some \; p > 0 \Big\}$, where $□ = (□_{r,u,j})$ be a triple sequence.

2. Main Results.

Proposition.2.2. Let $F = (F_1, F_2)$be a double Orlicz-functions satisfy the Δ_2_conditions, then

$i)\ \Omega^3(F_2, \Delta, q) \subset \Omega^3(F_1, \Delta, q)$ for $\Omega^3 = 3l_\infty, 3c, 3c_0$ if $F_2\ (x, y) \leq F_1\ (x, y)$

$\forall (x, y) \in [0, \infty) \times [0, \infty)$.

$ii)\ \Omega^3(F_1, \Delta, q) \subset \Omega^3(F \circ F_1, \Delta, q)$ for $\Omega^3 = 3l_\infty, 3c, 3c_0$.

Proof. (i) The proof is obvious.

(ii) Consider $\Omega^3 = 3c$. Let $(x, y) \in 3c(\Delta, F_1, q)$.

Then for some $\rho > 0$,

$3c\ (\Delta, F, q) =$

$$\Big\{ (x, y) \in \Omega^3 : p - \lim_{r,u,j\to\infty} \left[F_1 \left(q \left(\frac{|\Delta x_{r,u},□ - l_1|}{\rho} \right) \right) □\ F_2 \left(q \left(\frac{|\Delta y_{r,u},□ - l_2|}{\rho} \right) \right) \right]$$

$= 0 \; for \; some \; \rho > 0 \Big\}$,where

$$F_1 \left(q \left(\frac{|\Delta x_{r,u},□ - l_1|}{\rho} \right) \right) \to 0, as\ r, u, j \to \infty$$

$$F_2 \left(q \left(\frac{|\Delta y_{r,u},□ - l_2|}{\rho} \right) \right) \to 0\ as\ r, u, j \to \infty.$$

By (i) we get $[F\ (x, y) \leq F_1\ (x, y)]$.

Then $F \left[F_1 \left(q \left(\frac{|\Delta x_{r,u},□ - l_1|}{\rho} \right) \right) \right] \to 0, as\ r, u, j \to \infty$.

$$(F \circ F_1) \left(q \left(\frac{|\Delta x_{r,u},□ - l_1|}{\rho} \right) \right) \to 0, as\ \ r, u, j \to \infty.$$

Hence $(x, y) \in \Omega^3(F \circ F_1, \Delta, q)$. We can be proved the spaces $3l_\infty, 3c_0$ by a similar way.

This complete the prove.

Theorem2.3.The space $\Omega^3(\Delta_n^v, F, p, q, \varphi)$ are paranormed space , paranormed by

$g(x,y) = \sum_{r,u,j=1}^{m,l,n} |x_{r,u,j} + y_{r,u,j}| \quad +$

$inf\left\{ \rho^{\frac{p_{r,u,j}}{H}} : sup_{r,u,j} \left[F_1 \left(q\left(\frac{|\Delta^v x_{r,u,j}|}{\rho} \right) \right) ⍰ F_{2_r} \left(q\left(\frac{|\Delta^v y_{r,u,j}|}{\rho} \right) \right) \right] \leq 1 \right\}$,

where $H = max\left(1, sup_{r,u,j} p_{r,u,j}\right)$, and $\Omega^3 = 3l_\infty, 3c, 3c_0$.

Proof. We prove this theorem for the space $3l_\infty(\Delta_u^v, F, p, q, \varphi)$and the spaces $3c_0, 3c$ proved with a similar way.

Clearly $g(-x) = g(x), g(-y) = g(y)$,

let $(x_{r,u,j})$and $(y_{r,u,j})$ be any two triple sequences belong to any one of the spaces $\Omega^3(\Delta_n^v, F, P, q, \varphi)$, for $\Omega^3 = 3l_\infty, 3c$ and $3c_0$ .Then we have $\rho_1, \rho_2 > 0$ such that

$$sup_{r,u,j} F_1\left(\frac{q|\Delta_n^v\ x_{r,u,j}|}{\rho_1}\right) \leq 1$$

And

$$sup_{r,u,j} F_2(\frac{q|\Delta_n^v\ y_{r,u,j}|}{\rho_2}) \leq 1$$

Let $\rho = \rho_1 + \rho_2$.Then by convexity of F ,we have

$$sup_{r,u,j} F\left(\frac{q|\Delta_n^v(x_{r,u,j} + y_{r,u,j})|}{\rho}\right) \leq \left(\frac{\rho_1}{\rho_1 + \rho_2}\right) Sup_{r,u,j} F_1\left(\frac{q|\Delta_n^v x_{r,u,j}|}{\rho_1}\right)$$
$$+\left(\frac{\rho_2}{\rho_1 + \rho_2}\right) Sup_{r,u,j} F_2\left(\frac{q|\Delta_n^v y_{r,u,j}|}{\rho_1}\right) \leq 1.$$

Hence we have

$$g(x+y) = \sum_{r,u,j}^{m,l,n} |x_{r,u,j} + y_{r,u,j}| + inf\{\rho^{\frac{p_{r,u,j}}{H}} : Sup_{r,u,j} F(\frac{q|\Delta_n^v\ (x_{r,u,j} + y_{r,u,j})|}{\rho}) \leq 1\}$$
$$\leq \sum_{r,u,j=1}^{m,l,n} |x_{r,u,j}| + inf\{\rho_1^{\frac{p_{r,u,j}}{H}} : Sup_{r,u,j} F_1(\frac{q|\Delta_n^v\ x_{r,u,j}|}{\rho_1})\}$$
$$+ \sum_{r,u,j=1}^{m,l,n} |y_{r,u,j}| + inf\{\rho_1^{\frac{p_{r,u,j}}{H}} : Sup_{r,u,j} F_2(\frac{q|\Delta_n^v\ y_{r,u,j}|}{\rho_2}) \leq 1\}$$

This implies that

$g(x+y) \leq g(x) + g(y)$.

The continuity of the scalar multiplication follows from the following inequality:

$g(k'x) = \left\{ \sum_{r,u,j=1}^{m,l,n} |k'^{x_{r,u,j}}| + \inf\{\rho_1^{\frac{p_{r,u,j}}{H}} : Sup_{r,u,j} F_1\left(\frac{q|\Delta_n^v\ k' x_{r,u,j}|}{\rho_1}\right) + \right.$

$\left. \sum_{r,u,j=1}^{m,l,n} |k' y_{r,u,j}| + inf\{\rho_2^{\frac{p_{r,u,j}}{H}} : Sup_{r,u,j} F_2(\frac{q|\Delta_n^v\ k' y_{r,u,j}|}{\rho_1}) \right\} \leq 1$

$= |k'| \sum_{r,u,j=1}^{m,l,n} |x_{r,u,j}| + \inf\{(T|k'|)^{\frac{p_{r,u,j}}{H}} : Sup_{r,u,j} F_1 \left(\frac{q|\Delta_n^v \ x_{r,u,j}|}{\rho_1}\right) + |k'| \sum_{r,u,j=1}^{m,l,n} |y_{r,u,j}| +$

$inf\{(T|k'|)^{\frac{p_{r,u,j}}{H}} : Sup_{r,u,j} F_2 \left(\frac{q|\Delta_n^v \ y_{r,u,j}|}{\rho_1}\right) \} \leq 1.$

Hence the spaces $\Omega^3(\Delta_n^v, F, p, q, \varphi)$ for $\Omega^3 = 3l_\infty, 3c_0 and\ 3c$ are paranormed spaces.

■

Theorem.2.4. Suppose a semi−normed space (X^2, q) which is complete,where $\Omega^3 = 3l_\infty, 3c, 3c_0$ are complete semi−normed spaces semi−normed by

$\varphi(x, y) = inf \left\{\rho > 0 : sup_{r,u,j} \left[F_1 \left(\frac{q|\Delta^v \ x_{r,u,j}|}{\rho_1}\right) ⊕ F_2 \left(\frac{q|\Delta^v \ y_{r,u,j}|}{\rho_1}\right) \right] \leq 1 \right\}$, where

$F_1 \left(\frac{q|\Delta^v \ x_{r,u,j}|}{\rho_1}\right) \leq 1$, and $F_2 \left(\frac{q|\Delta^v \ y_{r,u,j}|}{\rho_1}\right) \leq 1.$

Proof.

Let $(x_{r,u,j}^i, y_{r,u,j}^i)$ be any triple Cauchy sequence in $3\ell_\infty(\Delta_n^v, F, q)$ where x^i , y^i is a Cauchy sequence, such that $(x_{r,u,j}^i)$ and $(y_{r,u,j}^i)$ be a triple Cauchy sequence in $3\ell_\infty(\Delta_n^v, F_1, q)$, $3\ell_\infty (\Delta_n^v, F_2, q)$ respectively.

Let fixed numbers $m_1, m_2 > 0$, then for all $\frac{\epsilon}{m_1 m_2} > 0$, ∃ a positive integers N such that $(\|x^i - x^t\|_{\Delta_n^v} , \| y^i - y^t \|_{\Delta_n^v}) < \frac{\epsilon}{m_1 m_2}, for\ all\ i, t \geq N, also\ for\ m_2 > 0$, choose $F(\frac{m_1 m_2}{2}) \geq 1$

Using the definition of seminorm , we have ,

$$\left[sup_{r,u,j} \left[\left(F_1 \left(\frac{q|\Delta_n^v x^i_{r,u,j} - \Delta_n^v x^t_{r,u,j} |}{\varphi_{(x,y)}(x^i - x^t)}\right)\right) ⊕ \left(F_2 \left(\frac{q \left|\Delta_n^v y^i_{r,u,j} - \Delta_n^v y^t_{r,u,j} \right|}{\varphi_{(x,y)}(y^i - y^t)}\right)\right) \right] \right] \leq F \left(\frac{m_1 m_2}{2}\right),$$

$for\ all\ i, t \geq N$, where

$$\left[sup_{r,u,j} \left[q \left(F_1 \left(\frac{|\Delta_n^v x^i_{r,u,j} - \Delta_n^v x^t_{r,u,j} |}{\varphi_{(x,y)}(x^i - x^t)}\right)\right) \right] \leq F \left(\frac{m_1 m_2}{2}\right),\right.$$

$$\left[q \left(F_2 \left(\frac{\left|\Delta_n^v y^i_{r,u,j} - \Delta_n^v y^t_{r,u,j} \right|}{\varphi_{(x,y)}(y^i - y^t)}\right)\right) \right] \Bigg] \leq F \left(\frac{m_1 m_2}{2}\right). \Bigg[sup_{r,u,j}$$

So,

$$q \left[\left(F_1 \left(\frac{\left|\Delta_n^v \ x^i_{r,u,j} - \Delta_n^v \ x^t_{r,u,j}\right|}{\varphi_{(x,y)}(x^i - x^t)}\right)\right) ⊕ \left(F_2 \left(\frac{\left|\Delta_n^v y^i_{r,u,j} - \Delta_n^v y^t_{r,u,j} \right|}{\varphi_{(x,y)}(y^i - y^t)}\right)\right) \right] \leq 1 \text{, such that}$$

$$q \left[\left(F_1 \left(\frac{\left| \Delta_n^v \ x^i_{r,u,j} - \Delta_n^v \ x^t_{r,u,j} \right|}{\varphi_{(x,y)}(x^i - x^t)}\right) \right) ⊕ \left(F_2 \left(\frac{\left|\Delta_n^v y^i_{r,u,j} - \Delta_n^v y^t_{r,u,j} \right|}{\varphi_{(x,y)}(y^i - y^t)}\right) \right) \right] \leq$$

$$q\left[(F_1 (\frac{m_1 m_2}{2})) \,(F_2 (\frac{m_1 m_2}{2})) \right] \geq 1.$$

The implies that

$$\left|(\Delta_u^v x^i{}_{r,u,} - \Delta_u^v x^t{}_{r,u,} , \Delta_u^v y^i{}_{r,u,} - \Delta_u^v y^t{}_{r,u,}) \right| \leq$$

$\frac{1}{2} m_1 m_2 \times \frac{1}{m_1 m_2} \epsilon = \frac{1}{2}\epsilon$, for all r, u and j we get

$$\left| \Delta_n^v x^i{}_{r,u,j} - \Delta_n^v x^t{}_{r,u,j} , \Delta_n^v y^i{}_{r,u,j} - \Delta_n^v y^t{}_{r,u,j} \right| \leq \frac{\epsilon}{2} , for\ all\ i , t \geq N .$$

Hence $(\Delta^v x^i_{r,u,j})$, $(\Delta^v y^i_{r,u,j})$ are triple Cauchy sequence in $\mathbb{R}$ such that $(\Delta_n^v x^i_{r,u,j} , \Delta_n^v y^i_{r,u,j})$ atriple Cauchy sequence in $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$, such that

$(\Delta^v x^i{}_{r,u,j})$, $(\Delta^v y^i{}_{r,u,j})$ are triple Cauchy sequence in $3\ell_\infty (\Delta_u^v, F_1 , q)$ and

$3\ell_\infty(\Delta_u^v, F_1 , q)$.Therefore each $0 < \epsilon < 1$),there exist a positive integer N such that

$$\left|(\Delta^v x^i_{r,u,j} - \Delta^v x^t_{r,u,j} , \Delta^v y^i_{r,u,j} - \Delta^v y^t_{r,u,j}) \right| < \epsilon\ for\ all\ i, t \geq N$$

Now ,using the continuity of F_1 , F_2 for each r, u we get :

$$sup_{r,u,j \geq N} \left[q\left[(F_1 (\frac{\Delta_n^v x^i{}_{r,u,} - \lim_{t\to\infty} \Delta_n^v x^i{}_{r,u,}}{\rho})) (F_2 (\frac{\Delta_n^v y^i{}_{r,u,} - \lim_{t\to\infty} \Delta_n^v y^i{}_{r,u,}}{\rho})) \right]\right] \leq 1.$$

Thus

$$sup_{r,u,j \geq N} \left[q \left[(F_1 (\frac{|\Delta_n^v x^i{}_{r,u,} - \Delta_n^v x{}_{r,u,}|}{\rho})) (F_2 (\frac{|\Delta_n^v y^i{}_{r,u,} - \Delta_n^v y{}_{r,u,}|}{\rho})) \right] \right] \leq 1$$

Taking infimum of $\rho^{,s}$ we have

$$inf \left\{ \rho > 0 : \left[sup_{r,u,j \geq N}\, q\left[(F_1 (\frac{|\Delta_n^v x^i{}_{r,u,} - \Delta_n^v x{}_{r,u,}|}{\rho})) \quad (F_2 (\frac{|\Delta_n^v y^i{}_{r,u,} - \Delta_n^v y{}_{r,u,}|}{\rho})) \right]\right] \leq 1 \right\} \leq \epsilon\ for\ all\ i \geq N\ and\ t \to \infty$$

Since $(x^i, y^i) \in 3\ell_\infty(\Delta_n^v , F, \mathrm{q})$ and F_1 , F_2 be an a double Orlicz functions , then

$F = (F_1 , F_2)$ is an a double Orlicz functions for each r, u and by continuous , we get that $(x , y) \in 3\ell_\infty (\Delta_n^v , F, \mathrm{q})$is linear .Then $3\ell_\infty (\Delta_n^v , F, \mathrm{q})$ is seminorm.

The rest of proof $3c$, $3c_0$ is like the previous case $3l_\infty$.

Conflict of Interests.
There are non-conflicts of interest

Reference.

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الخلاصة

الهدف الرئيسي من هذا البحث هو انتاج فضاء شبه معياري مؤثر لاختلاف فضاءات المتتابعات الثلاثية باستخدام دالة اورليسز المضاعفة, وندرس بعض الخصائص المختلفة مثل الكمال , الصلابة , الرتابة, التناظر وغيرها .