Studying the Isotopes nearby Closed Shell of Kr, Xe and Hg Using Interacting Boson Models

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Abstract

The closed shell of neutrons or protons means there is no boson numbers, which are important for applying IBM models. The properties of nuclear structure for isotopes depend on the boson numbers. If they are small the vibrational properties will appear or nearby from these properties. our theoretical results using two models (IBM-1 and IBM-2) are acceptable matching regard to the energy levels with the experimental data, electric transitions and potential energy surface. For neutron number ^{82,84}Kr is nearby 50 closed shell which has transition properties between U(5) and O(6), ^{132,134}Xe is nearby 80 has U(5) properties and nearby 126 but for ^{202,204}Hg have different properties.

Keywords: Interacting boson models, energy levels, electric transitions and potential energy surface.

Introduction

When protons and/or neutrons are filled from the lowest to the higher-lying orbitals to reach special values like 2, 8, 20, 28, 50, 82, 126, …, then a nucleus is stable and hence big amount of energy is needed to excite the nucleus from the closed shell to the next. Magic numbers were called for these numbers, which become evident as a sudden drop of the observed nucleon separation energies. In exotic nuclei, conventional magic numbers may become no longer valid, even giving rise to novel shell structures not heretofore recognized[1].

In 1974, a new nuclear model was proposed by Arima and Iachello, which called (IBM) interacting boson model of nuclear structure. To correlate the collective properties of odd-even nuclei, the IBM has been applied by coupling the fermion as a single-particle to the even-even core and even-even nuclei[2], [3].

IBM-1 is abbreviate to the original version of the interacting boson model, and it is applicable to even-even nuclei. The fermion states which cannot be represented are single-particle excitations, and high-angular momentum, low-seniority states[4]. Collective fermion states are well reproduced. The IBM-1 does not distinguishing between bosons connected with proton-proton and neutron-neutron pairs (this is done in an extended version of the model. The IBM-2, which is description of collective excitations) and does not consider bosons connected with mixed proton-neutron pairs[5]. In the IBM-1, the bosons number N is calculated by summation the protons and neutrons numbers as: $N = N_\pi + N_\nu$ [6], but in IBM-2, the bosons number is calculated as N_π and N_ν severally.

The Hamiltonians

IBM-1, describes the low-lying collective excitations of an even-even nucleus as terms of the s (L=0) and d (L=2) bosons. For a fixed boson number N, only one of the one-body term and five of the two body terms are independent, as it can be seen by noting $N = n_s + n_d$ [7]. The IBM-1 Hamiltonian can be expressed as[8]:

$$\hat{H} = E\hat{n}_d + a_0\hat{p}.\hat{p} + a_1\hat{L}.\hat{L} + a_2\hat{Q}.\hat{Q} + a_3\hat{T}_3.\hat{T}_3 + a_4\hat{T}_4.\hat{T}_4 \quad .....1$$

$\hat{n}_d = \left(d^\dagger.d\right)$ is the total number of d-boson operator. $\hat{p} = 1/2\left[\left(d.d\right) - \left(s.s\right)\right]$ is the pairing operator. $\hat{L} = \sqrt{10}\left[d^\dagger \times d\right]^{(1)}$ is the angular momentum operator. $\hat{Q} = \left[d^\dagger \times s + s^\dagger \times d\right]^{(2)} + \chi\left[d^\dagger \times d\right]^{(2)}$ is the quadruple operator. χ is the parameter of quadrupole structure (between 0 and $\pm\frac{\sqrt{7}}{2}$). $\hat{T}_m = \left[d^\dagger \times d\right]^m$ is the octoupole (m=3) and hexadecapole (m=4) operator and $E = E_d - E_s$ is the boson energy. The parameters a_0, a_1, a_2, a_3 and a_4 are the strength of the pairing, angular momentum, quadruple, octoupole and hexadecapole interaction between the bosons, respectively.

IBM-2 Hamiltonian is[9],[10]:

$$\hat{H} = E_d\left(\hat{n}_{d\nu} + \hat{n}_{d\pi}\right) + \kappa\left(\hat{Q}_\nu.\hat{Q}_\pi\right) + V_{\nu\nu} + V_{\pi\pi} + M_{\nu\pi} \quad ...... 2$$

E_d: the energy difference between s and d boson, $n_{d\rho}$ represents the number of d bosons, where ρ goes along with π (proton) or ν (neutron) bosons, the second term refers to the quadruple – quadruple interaction between proton and neutron with strength κ, where the quadruple operator Q_ρ can be written as:

$$Q_\rho = \left[d_\rho^\dagger s_\rho + s_\rho^\dagger d_\rho\right]^{(2)} + \chi_\rho\left[d_\rho^\dagger d_\rho\right]^{(2)} \quad ...... 3$$

where χ_ρ is the quadruple deformation parameter for proton and neutron. The $V_{\pi\pi}$ and $V_{\nu\nu}$, which refer to the interaction between like boson, are sometimes present to improve the fit to experimental energy spectra and they are given by:

$$V_{\rho\rho} = \frac{1}{2}\sum_{L=0,2,4} C_L^\rho\left(\left[d_\rho^\dagger d_\rho^\dagger\right]^{(L)} . \left[d_\rho d_\rho\right]\right) \quad ...... 4$$

The last term in equation (2) is Majorana term $M_{\nu\pi}$ has the parameters of and ξ_2, ξ_1 :[11]as ξ_3

$$M_{\nu\pi} = \frac{1}{2}\xi_2\left(\left[s_\nu^\dagger d_\pi^\dagger - d_\nu^\dagger s_\pi^\dagger\right]^{(2)} . \left[s_\nu d_\pi - d_\nu s_\pi\right]^{(2)}\right) - \sum_{k=1,3}\xi_k\left(\left[d_\nu^\dagger d_\pi^\dagger\right]^k . \left[d_\nu d_\pi\right]^k\right) \quad ..... 5$$

Electric transitions

Only the protons should appear in the description of electromagnetic transitions, since they alone carry charge inside the nucleus. On the one hand, the interactions between the nucleons may exchange charge and thus contribute to the current, and on the other hand, neutrons and protons are coupled by center of mass conservation[12]. The absolute transition rates not only are a sensitive property of nuclear structure but also provide a stringent test for various models. Most *B(E2)* values known to date were measured by coulomb excitation[13]. Then the electric quadruple transition in IBM-1 is[14]:

$$\begin{aligned} T_m^{E2} &= \alpha_2 \left[d^\dagger \times s + s^\dagger \times d\right]_m^2 + \beta_2 \left[d^\dagger \times d\right]_m^2 \\ &= \alpha_2 \left(\left[d^\dagger \times s + s^\dagger \times d\right]_m^2 + \chi \left[d^\dagger \times d\right]_m^2\right) = e_B \hat{Q} \end{aligned} \quad \text{...... 6}$$

where $\alpha_2 = e_B$ (effective charge) and $\beta_2 = \chi\alpha_2$ and for E2 transition in IBM-2[15]:

$$\begin{aligned} \hat{T}^{(E2)} &= \hat{T}_\pi^{(E2)} + \hat{T}_\nu^{(E2)} \\ \hat{T}_\pi^{(E2)} &= e_\pi Q_\pi \ , \ \hat{T}_\nu^{(E2)} = e_\nu Q_\nu \end{aligned} \quad \text{...... 7}$$

e_π, e_ν: Stand for the effective charge for each of the proton and the neutron, the unity (eb) is dependent on the number of bosons protons and neutrons (N_π , N_ν).

Potential energy surface

The energy surface, as a function of β and γ, has been given by[16]:

$$PES(N,\beta,\gamma) = \frac{NE_d}{(1+\beta^2)} + \frac{N(N+1)}{(1+\beta^2)^2}(A_1\beta^4 + A_2\beta^3\cos(3\gamma) + A_3\beta^2 + A_4 \quad \text{...... 8}$$

where the A_i's are coefficients.

β: a measure of the total deformation of nucleus, when $\beta = 0$ the shape is spherical, and be distorted when $\beta \neq 0$, and γ is the amount of deviation from the symmetry and correlates with the nucleus, if γ=0 the shape is prolate, and if γ=60 the shape becomes oblate[17],[18]. The following equations represented potential energy surface for three dynamical symmetries[19]:

$$E(N,\beta,\gamma) \propto \begin{cases} U(5): & \varepsilon_d N \dfrac{\beta^2}{1+\beta^2} \\ SU(3): & KN(N-1)\dfrac{\frac{3}{4}\beta^4 - \sqrt{2}\beta^3\cos 3\gamma + 1}{\left(1+\beta^2\right)^2} \\ O(6): & K'N(N-1)\left(\dfrac{1-\beta^2}{1+\beta^2}\right)^2 \end{cases} \quad \text{...... 9}$$

where $K \propto a_2$ and $\acute{K} \propto a_0$ in equation 1.

Results and Discussion

Determining the parameters of the Hamiltonian depend on the ratios of experimental energy levels [20],[21]. Firstly the ratio of $E4_1/E2_1$=2, 2.5 and 3.33, secondly the ratio of $E6_1/E2_1$=3, 4.5 and 7, finally the ratio of $E0_2/E2_1$=2, >>2 and 4.5 for U(5), SU(3) and O(6) limits respectively.

From these ratios we can estimate the parameters of the two Hamiltonian in two models for these isotopes. The parameters in equation 1 were used to get fitting of the energy levels as tables 1 for IBM-1 model. In IBM-2 the parameters in equation 2 were represented in table 2. From the results in these tables, figures 1 to 6 were been drawn for our isotopes. These figures represent a reasonable matching between the models with the experimental data. This is denoted that the properties for the ^{82,84}Kr is closer to U(5) limit because the parameter E in two models is the biggest. A good example for vibrational properties is ^{132,134}Xe isotopes but there are some different properties for ^{202,204}Hg which explained clearly by Gh. Jaber and M. Muttaleb[22].

Electric transition can be calculated applying the equations 6 and 7 for the two models and the results represent in table 3 for some selection transitions. From table 3, B(E2) for the first transition ($2_1 \rightarrow 0_1$) decreases with decrease of boson number because nearby from stability. The same behavior for the others transitions with some little differences.

The last step to investigate the nuclear structure for the isotopes is the potential energy surface applying equations 8 and 9, which represent in figures 7 to 12. These figures have the symmetric shapes on the right and the contour lines on the left of these figures. Figures 7 and 8 represent the decreasing in the potential for ^{84}Kr more than for ^{82}Kr because the decreasing in bosons nearby closed shell 50 for neutron. There is small deviation in contour lines and accumulated between β =0.5 and 1.

There is no deformation in figures 9 and 10, because the closer from magic number 80 for neutrons with bid decreasing in the potential for ^{134}Xe. There is decreasing in the potential for ^{204}Hg more than ^{202}Hg. There is deformation with nearby the closed shell or magic number 126 for neutrons of ^{204}Hg isotope. This is because the mercury is rich with neutrons and the fitting get it with applying the parameter of the strength of quadruple.

Table 1: IBM-1 model Hamiltonian parameters

The isotopes	Boson Number	E MeV	a_0 MeV	a_1 MeV	a_2 MeV	a_3 MeV	a_4 MeV	χ	α_2 eb	β_2 eb
^{82}Kr	6	0.778	0.0511	0.026	0	-0.0243	0	0	0.08	-0.06
^{84}Kr	5	0.92572	0.094	0.0141	0	0.03183	0	0	0.07	-0.05
^{132}Xe	4	0.28324	0	0.011746	0	-0.2	0.33	0	0.15	-0.1
^{134}Xe	3	0.8793	0	0.0047	0	-0.0213	-0.017	0	0.14	-0.1
^{202}Hg	3	0	0.36399	0.0281	0	0.2044	-0.06	0	0.07	0
^{204}Hg	2	0	0.83551	0.01304	0.096	0.5433	0	-1	0.04	0

Table 2: IBM-1 model Hamiltonian parameters in MeV (χ unit less).

The isotopes	Neutron boson	Protons boson	E_d	k	χ_ν	χ_π	$\xi_{1=3}$	ξ_2	C_ν^L	C_π^L
^{82}Kr	2	4	0.9233	-0.06	0.52	-0.6	0.11	0.54	2.33, -1, 1.3	-0.6, 0.1, 0.3
^{84}Kr	1	4	0.9426	-0.04	0.54	-0.6	0.11	0.52	2.33, -1, 1.3	0.05, 0.18, 0.34
^{132}Xe	2	2	1.04	-0.5	-0.86	0.8	-0.6	0.15	-0.5, -1.6, -1.6	0,0,0
^{134}Xe	1	2	0.9524	-0.1	-0.73	0.8	-0.6	0.18	0, 0, 0,	0,0,0
^{202}Hg	2	1	0.4644	-0.26	0.4	-0.48	0.4	-0.14	0.14, -0.02,0.13	0,0,0
^{204}Hg	1	1	0.4606	-0.2	0.4	-0.48	0.4	-0.14	0, 0, 0	0,0,0

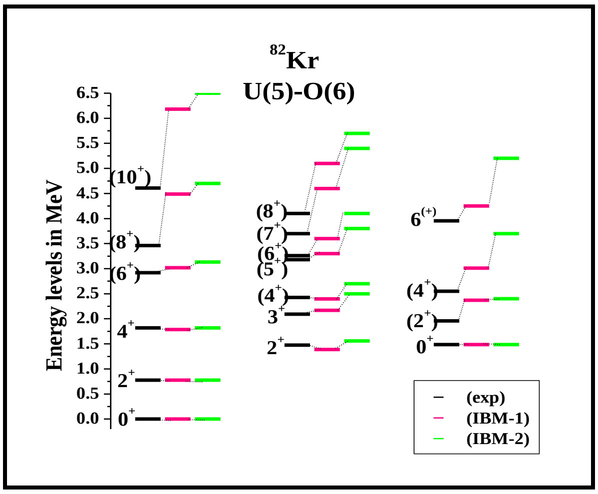

Figure 1: Experimental energy levels compared with IBM-1 and 2 for ^{82}Kr.

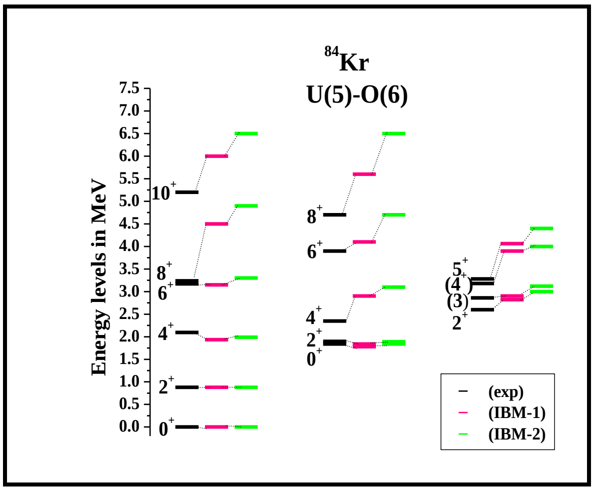

Figure 2: Experimental energy levels compared with IBM-1 and 2 for ^{84}Kr.

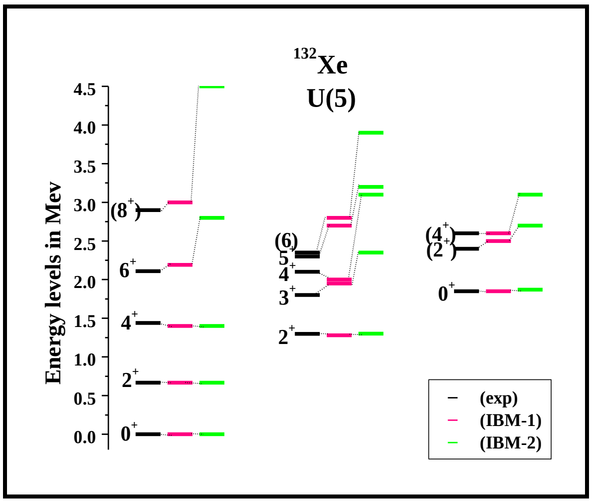

Figure 3: Experimental energy levels compared with IBM-1 and 2 for ^{132}Xe.

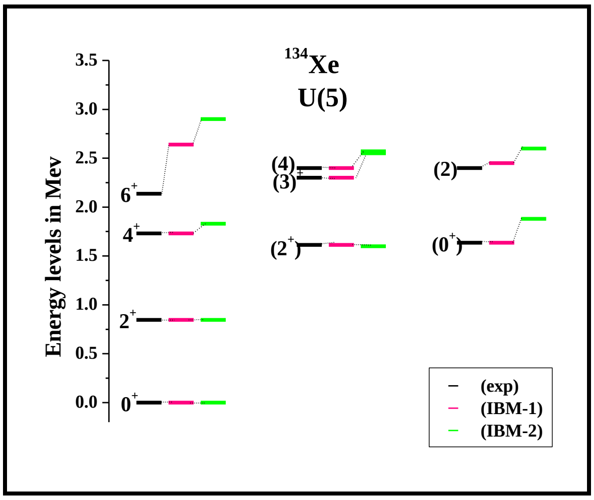

Figure 4: Experimental energy levels compared with IBM-1 and 2 for ^{134}Xe.

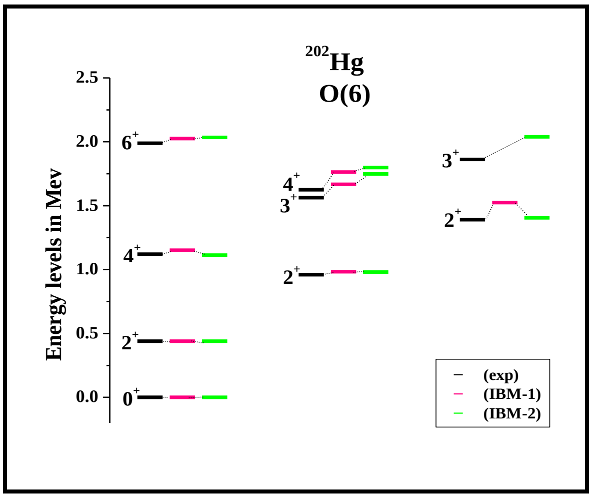

Figure 5: Experimental energy levels compared with IBM-1 and 2 for ^{202}Hg.

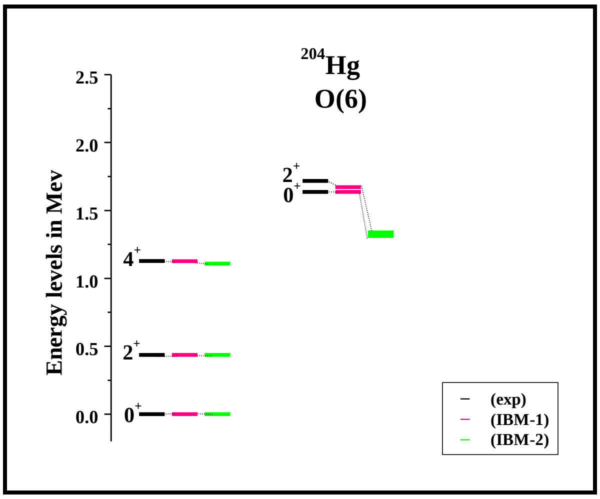

Figure 6: Experimental energy levels compared with IBM-1 and 2 for ^{204}Hg.

Table 3: The electric transitions (e^2b^2 unit) with positive parity for isotopes using IBM-1 and IBM-2 models.

The Isotope	$J_i \rightarrow J_f$	$2_1 \rightarrow 0_1$	$2_1 \rightarrow 0_2$	$2_2 \rightarrow 2_1$	$4_1 \rightarrow 2_1$	$4_2 \rightarrow 4_1$	$6_1 \rightarrow 4_1$	$6_2 \rightarrow 6_1$
^{82}Kr	exp	0.045	0.008	0.011	0.0677	0.0635	0.011	-
	IBM-1	0.046	0.012	0.074	0.074	-	-	-
	IBM-2	0.04	0.009	0.06	0.062	0	0	0.024
^{84}Kr	exp	0.026	-	0.028	0.03	0.015	0.015	-
	IBM-1	0.0276	0.006	0.0415	0.0415	-	-	-
	IBM-2	0.023	0.006	0.036	0.036	0.03	0.03	0.01
^{132}Xe	exp	0.092	-	0.164	0.114	-	-	-
	IBM-1	0.085	0	0.125	0.12	-	-	-
	IBM-2	0.1	0.001	0.134	0.097	0.05	0.0953	0.018
^{134}Xe	exp	0.06	-	-	0.047	-	-	-
	IBM-1	0.0544	0.0145	0.0725	0.07	-	-	-
	IBM-2	0.07	0.012	0.051	0.09	0.018	0.06	-
^{202}Hg	exp	0.122	-	0.039	0.186	-	0.176	-
	IBM-1	0.119	0.0005	0.129	0.129	-	-	-
	IBM-2	0.1217	0.001	0.195	0.19	0.086	0.167	0.0397
^{204}Hg	exp	0.085	-	-	0.12	-	0.143	-
	IBM-1	0.08	0.0006	0.057	0.067	-	-	-
	IBM-2	0.1	0.0002	0.089	0.084	-	-	-

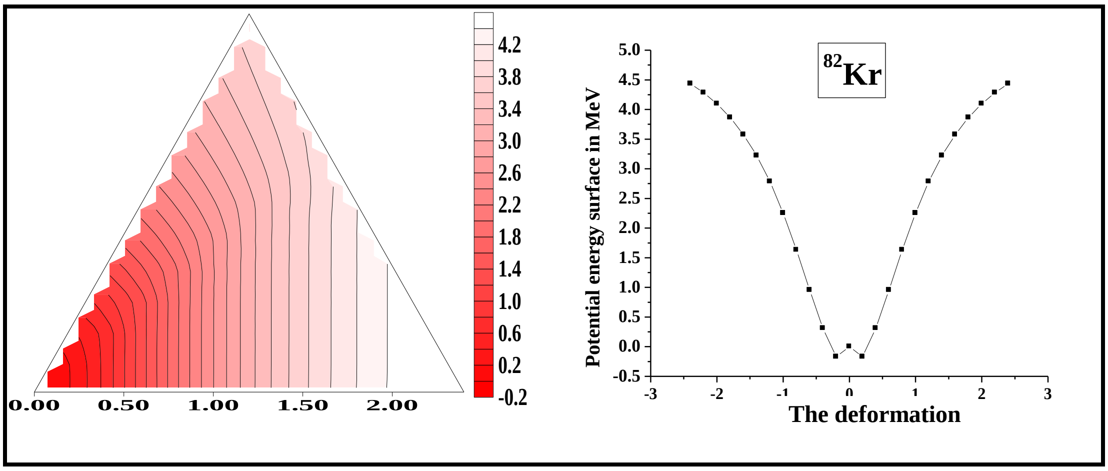

Figure 7: Potential energy surface with the deformation for ^{82}Kr using IBM-1.

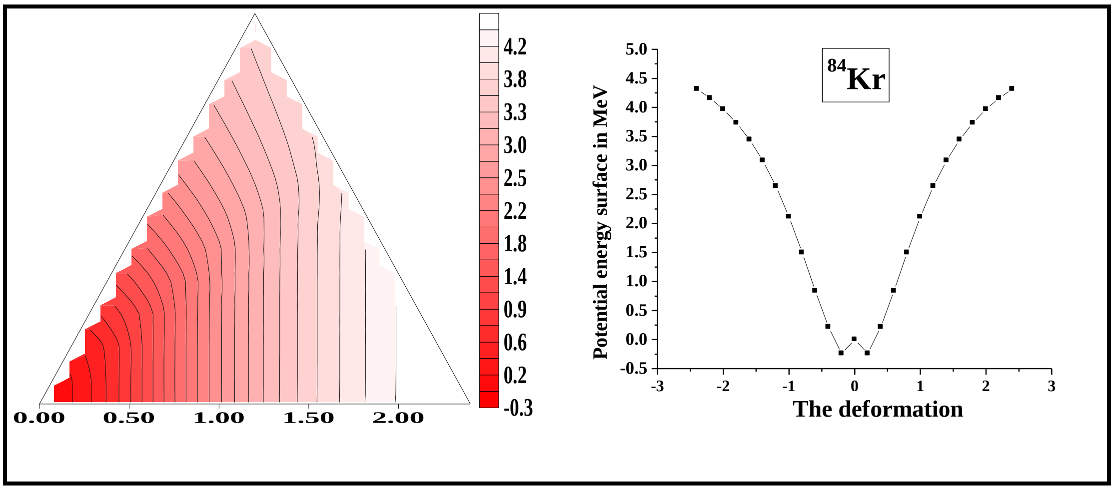

Figure 8: Potential energy surface with the deformation for ^{84}Kr using IBM-1.

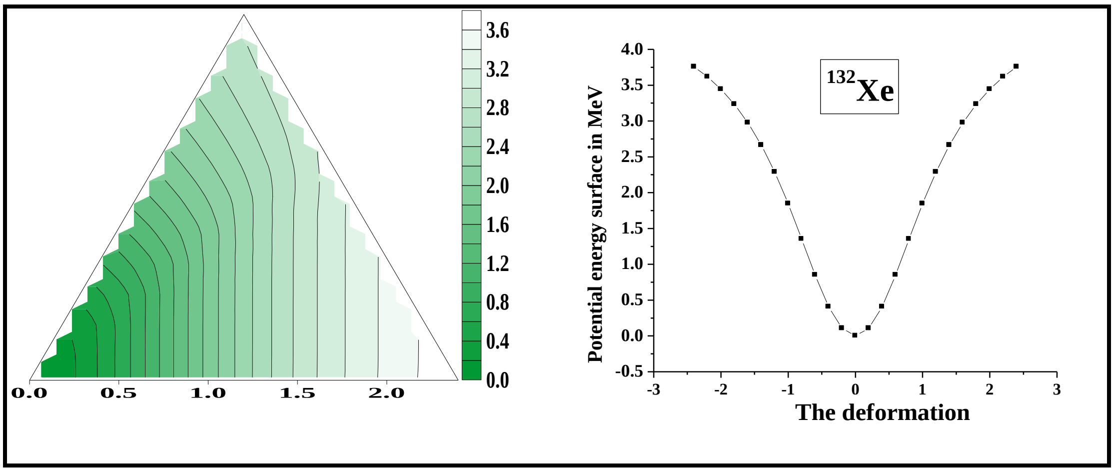

Figure 9: Potential energy surface with the deformation for ^{132}Xe using IBM-1.

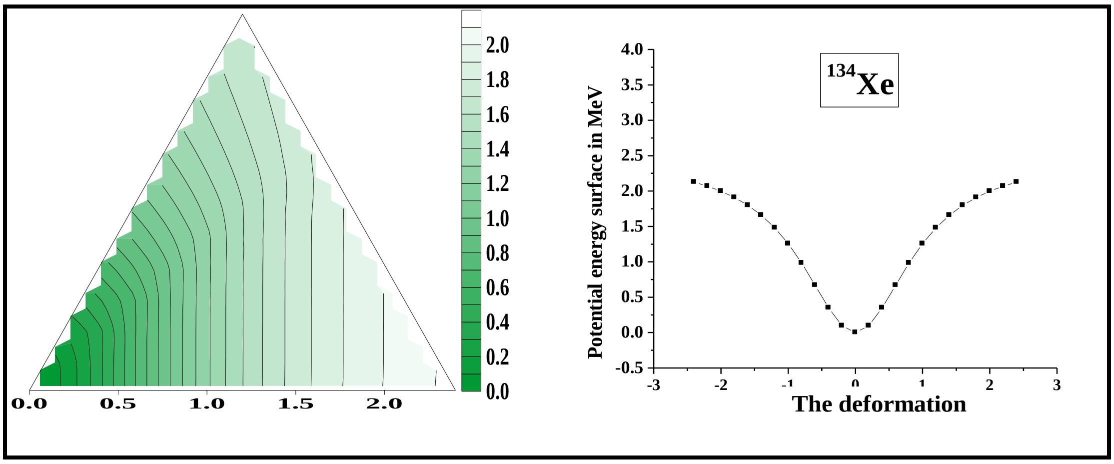

Figure 10: Potential energy surface with the deformation for ^{134}Xe using IBM-1.

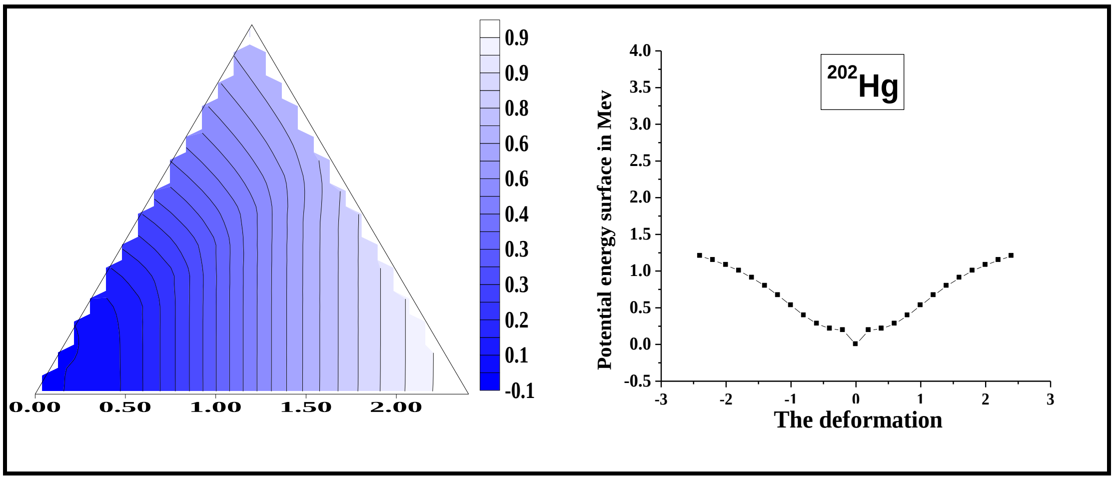

Figure 11: Potential energy surface with the deformation for ^{202}Hg using IBM-1.

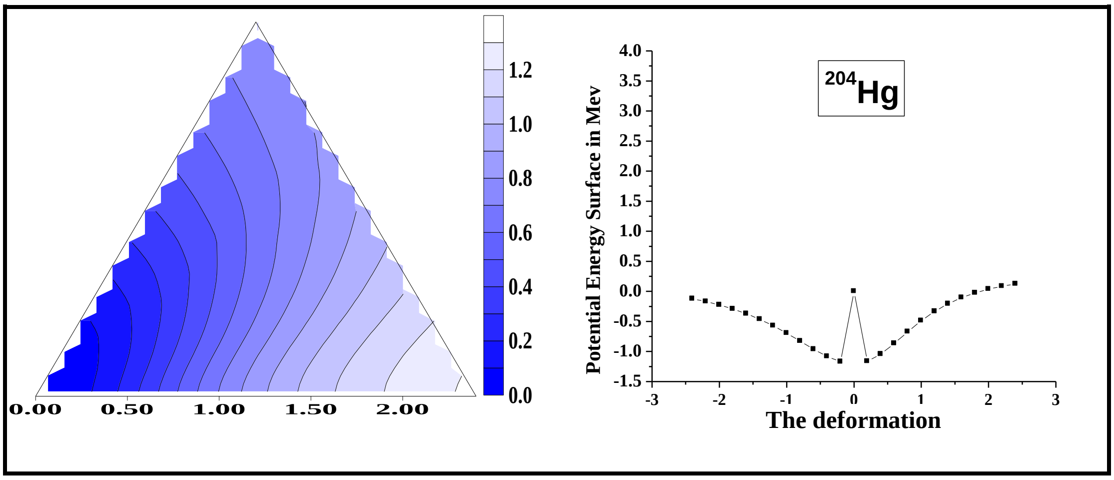

Figure 12: Potential energy surface with the deformation for ^{204}Hg using IBM-1.

Conclusions

The agreement between the results of the two models is very clear through the convergence of these results with the experimental results, especially for low levels. We can use the two models to get fitting with greater possibility using IBM-2. The kind of bosons (hole or particle) affect on the properties of the isotopes with small change in it. Calculations of B(E2) values show a good matching with the existing experimental results. These transitions denoted to some permission or forbidden transitions. Estimation the limit of the isotopes can be note from these electric transitions. However, there is difference between them, due to the effect of the deformation of these isotopes nuclei.

Potential energy surface is good for examine and emphasis the expected limit. Approaching the isotopes from closed shell, meaning that there is small deviation in contour lines. The potential distribute equally on the nuclei of these isotopes with symmetry in their wave function.

Conflict of Interests.
There are non-conflicts of interest

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الخلاصة

القشرة المغلقة لنيوترونات او للبروتونات تعني انه ليس هناك عدد من البوزونات التي تكون مهمة في تطبيق نماذج IBM. خواص التركيب النووي للنظائر يعتمد على عدد البوزونات، اذا كانت قليلة فان الخواص الاهتزازية سوف تظهر او تكن قريبة منها. النتائج النظرية من النموذجين IBM−1 و IBM−2 ذات تطابق جيد مع البيانات العملية لمستويات الطاقة، الانتقالات الكهربائية و سطح طاقة الجهد. عدد البروتونات ^{82,84}Kr قريبة من القشرة المغلقة 50 تمتلك خواص انتقالية بين U(5) و O(6)، ^{132,134}Xe قريبة من 82 تمتلك خواص U(5) والقريبة من 126 ولكن ل ^{202,204}Hg تمتلك خواص مختلفة.

الكلمات الدالة: نماذج البوزونات المتفاعلة، مستويات الطاقة، الانتقالات الكهربائية، سطح طاقة الجهد.