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Numerical Solutions of the Coupled Kortweg-de Vries Systems

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تقنية حسابية لحل أنظمة KDV المقترنة

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ABSTRACT

A Homotopy analytical method (HAM) for solving nonlinear coupled Korteweg-de Vries (KdV) system is presented. The HAM contains an assistant parameter h, which gives a helpful manner of controlling the merging locale of the arrangements. We will comparison exact solution with numerical results. This technique provides an efficient and effectives approximate analytical solution with high accuracy appeared graphically in figures demonstrate and affirm the tall precision and outlining the productivity and straightforwardness of the strategy.

Key words: Homotopy Analysis Method, Coupled KdV system, Analytical solution.

INTRODUCTION

The study of nonlinear partial differential equations problems has a significant effect on scientific fields, particularly in designing, science, strong state material science; plasma material science, optics, liquid mechanics, and chemical material science see **[1, 2, 3, 4, 7, 11, 19, 21 and 23].**

It is a set of nonlinear, dispersive differential equations with exact and precise solution. Boussinesq first proposed the KdV equation in 1877. Then later in 1895, Kortweg and De-Vries

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advanced the KdV equation to Russell's solutions model [14], such is low and restricted amplitude shallow-water waves in [20].

Kortweg-de Vries (KdV) equation has been a critical lesson of non-linear advancement conditions with various applications in the material science, engineering and biology. All these applications begin from a familiar physical demonstration and ended within KdV eq. We consider a particular restrain of physical issue. Meanwhile, the non-linear Schrödinger equation rolls a crucial part in fluctuations in deep and flow of water, whereas the KdV eq. portrays the impacts in superficial water. In blood plasma and material science, the KdV eq. delivers ion-acoustic arrangements [9]. Some researchers and analysts get symmetries since they can efficiently earn exact solutions to KdV equations [17 and 18].

Consider the following coupled KdV eq.'s given by [12]:

$$\begin{aligned} u_t - \alpha u_{xxx} - 6\alpha u u_x - \beta v v_x = 0, \\ v_t + v_{xxx} + 3u v_x = 0. \end{aligned} \tag{1}$$

Subject to

$$\begin{aligned} u(-L,t) = f_l(-L,t), \qquad v(-L,t) = g_l(-L,t), \\ v(L,t) = f_R(L,t), \qquad v(L,t) = g_R(L,t), \quad \forall\, t \geq 0. \end{aligned} \tag{2}$$

Since α and β are non-zero parameters, where $x \in [-L, L]$, f_l , f_R, g_l and $g_R$are functions to t. In equation (1), the derivation that, Hirota and Satsuma [12] to define iterations for a wave of water with diffusion and various relationships. During these years, authors attempt to use numerical solutions of two equations by make use of distinct methods. Trigonometric function transform [6], F-expansion [22], homotopy perturbation [8] and reduce differential transformation [10] are some selected methods. Many other ways used to get solutions Equation (1), [5 and 13] and references therein.

In this study, we use HAM to try to solve (1) numerically. HAM is well-known method for getting approximate solutions to system of PDEs in Equation, similar to other nonlinear analytical. In addition in the last section, many numerical examples are used to demonstrate the method's accuracy.

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THE ESSENTIAL CONCEPT OF THE HOMOTOPY ANALYSIS METHOD

This method was presented by using Homotopy, which is a topology concept [16]. According to HAM [15], let's have a look at the differential equation:

$$N[u(\tau)] = 0, \tag{3}$$

where N is a non-linear operator, τ denotes the presence of independent variables, we disregard all restrictions that cloud help us cope with it in a similar way. This means generalization of Homotopy method [15], such an equation can be expressed as a zero-order equation of deformation.

$$(1-p)L[\varphi(\tau;p) - u_0(\tau)] = phH(\tau)N[\varphi(\tau;p)]. \tag{4}$$

We define $p \in [0,1]$ to be the parameter for embedding, $h \neq 0$, is a non-zero auxiliary parameter and L is an auxiliary linear operator, $u_0(\tau)$is an educated guess of $u(\tau)$, $\varphi(\tau;p)$ is an unidentified function. $H(\tau) \neq 0$ is an auxiliary function, one must have substantial choice to take auxiliary items in HAM.

When $p = 0$, and $p = 1$ both,

$$\varphi(\tau;0) = u_0(\tau), \quad \varphi(\tau;1) = u(\tau).$$

hold. Therefore, as p rises from zero to one, the $\varphi(\tau;p)$ differs from the initial condition $u_0(\tau)$ to the solution $u(\tau)$. Expanding $\varphi(\tau;p)$ in Taylor series with regard to p, one has:

$$\varphi(\tau;p) = u_0(\tau) + \sum_{m=1}^{\infty} u_m(\tau)p^m,$$

where

$$u_m(\tau) = \frac{1}{m!}\frac{\partial^m \varphi(\tau;p)}{\partial p^m}\bigg|_{p=0}.$$

The auxiliary function $H(\tau)$, are all selected with such precision, the series comes to closed at $p = 1$, and then there's:

$$u(\tau) = u_0(\tau) + \sum_{m=1}^{\infty} u_m(\tau).$$

The following vector $\vec{u}_n$ will be defined as

$$\vec{u}_n = \{u_0(\tau), u_1(\tau), u_2(\tau), \dots, u_n(\tau)\}.$$

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Equation (4) to be Differentiated, m times in terms of the integrating parameter p, after that, let $p = 0$, and the mth-order displacement equation is derived by dividing all by m!, is obtained

$$L[u_m(\tau) - \chi_m u_{m-1}(\tau)] = hH(\tau)R_m(\vec{u}_{m-1}), \quad (5)$$

where

$$R_m(\vec{u}_{m-1}) = \frac{1}{(m-1)!}\frac{\partial^{m-1}\varphi(\tau;p)}{\partial p^{m-1}}\bigg|_{p=0}, \quad \chi_m = \begin{cases} 0, & m \le 1 \\ 1, & m > 1. \end{cases}$$

Using $L^{-1} = \int_0^t (\cdot)dt$ on both sides of the Equation (5), we obtained

$$u_m(\tau) = \chi_m u_{m-1}(\tau) + hL^{-1}[H(\tau)R_m(\vec{u}_{m-1})].$$

Thus, it's simple to obtain u_m for $m \ge 1$, we have m^{th}- order, as:

$$u(\tau) = \sum_{m=0}^{M} u_m(\tau).$$

When $M \to \infty$, shown that a precise approximation solution to the original Equation (3). For the above method's convergence with Liao's work. If Equation (3) allows for a unique solution, if Equation (3) fails to get a unique solution, among many other (possible) options, the HAM will provide one [24].

SOLVE THE COUPLED KDV SYSTEM USING HOMOTOPY ANALYSIS METHOD

In order to find HAM options of coupled KdV system (1), we opt for the auxiliary linear operators:

$$L[\varphi_1(x,t;p_1)] = \frac{\partial}{\partial t}[\varphi_1(x,t;p_1)],$$
$$L[\varphi_2(x,t;p_2)] = \frac{\partial}{\partial t}[\varphi_2(x,t;p_2)],$$

which satisfy

$$L[C_1 + tC_2] = 0,$$

The integral constants are $C_1(x)$ and $C_2(x)$. Now, a non-linear operator is defined as:

$$N_1[\varphi_1,\varphi_2] = \frac{\partial\varphi_1(x,t;p_1)}{\partial t} - \alpha\frac{\partial^3\varphi_1(x,t;p_1)}{\partial x^3} - 6\alpha\varphi_1(x,t;p_1)\frac{\partial\varphi_1(x,t;p_1)}{\partial x} - \beta\varphi_2(x,t;p_2)\frac{\partial\varphi_2(x,t;p_2)}{\partial x},$$

and

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$$N_2[\varphi_1,\varphi_2] = \frac{\partial \varphi_2(x,t;p_2)}{\partial t} + \frac{\partial^3 \varphi_2(x,t;p_2)}{\partial x^3} + 3\varphi_1(x,t;p_1)\frac{\partial \varphi_2(x,t;p_2)}{\partial x}$$

We structure the zeroth-order deformation equation using the given definition:

$$(1-p_1)L[\varphi_1(x,t;p_1) - u_0(x,t)] = p_1 hH(x,t;p_1)N_1[\varphi_1,\varphi_2], \qquad (6)$$

for $p_1 = 0$ and $p_1 = 1$, We have the ability to write

$$\varphi_1(x,t;0) = u_0(x,t), \qquad \varphi_1(x,t;1) = u(x,t), \qquad (7)$$

$$(1-p_2)L[\varphi_2(x,t;p_2) - v_0(x,t)] = p_2 hH(x,t;p_2)N_2[\varphi_1,\varphi_2]. \qquad (8)$$

For $p_2 = 0$ and $p_2 = 1$, We have the ability to write

$$\varphi_2(x,t;0) = v_0(x,t), \qquad \varphi_2(x,t;1) = v(x,t). \qquad (9)$$

As p_1 and p_2 increases from 0 to 1, $\varphi_1(x,t;p_1)$ and $\varphi_2(x,t;p_2)$ differ from $u_0(x,t)$, $v_0(x,t)$ to the exact solutions $u(x,t)$ and $v(x,t)$. As a result of Taylor's theorem, equations (7) and (9), express as follows:

$$\varphi_1(x,t;p_1) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t)p_1^m, \qquad (10)$$

$$\varphi_2(x,t;p_2) = v_0(x,t) + \sum_{m=1}^{\infty} v_m(x,t)p_2^m, \qquad (11)$$

where

$$u_m(x,t) = \frac{1}{m!}\frac{\partial^m \varphi_1(x,t;p_1)}{\partial p_1^m}\bigg|_{p_1=0}, \quad v_m(x,t) = \frac{1}{m!}\frac{\partial^m \varphi_2(x,t;p_2)}{\partial p_2^m}\bigg|_{p_2=0}.$$

The linear operator L with initial guess, and the auxiliary parameters h are all determined, the series of Equations (10) and (11) are convergent, at $p_1 = 1$ and $p_2 = 1$, and then we have:

$$u(x,t) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t), \qquad (12)$$

$$v(x,t) = v_0(x,t) + \sum_{m=1}^{\infty} v_m(x,t). \qquad (13)$$

Now, we define the vectors

$$\vec{u}_n = \{u_0(x,t), u_1(x,t), u_2(x,t), \dots, u_n(x,t)\},$$
$$\vec{v}_n = \{v_0(x,t), v_1(x,t), v_2(x,t), \dots, v_n(x,t)\}.$$

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The zeroth-order deformation Equations (6) and (8) are differentiated, m times in relation to p_1 and p_2, then dividing by $m!$, and lastly establishing, $p_1 = 0$ and $p_2 = 0$, we get the following m^{th}-order equations of displacement:

$$L[u_m(x,t) - \chi_m u_{m-1}(x,t)] = hH(x,t)R_{1m}(\vec{u}_{m-1}, \vec{v}_{m-1}), \quad (14)$$

$$L[v_m(x,t) - \chi_m v_{m-1}(x,t)] = hH(x,t)R_{2m}(\vec{u}_{m-1}, \vec{v}_{m-1}), \quad (15)$$

given the conditions

$$u_m(x,0) = 0, \qquad v_m(x,0) = 0,$$

where

$$R_{1m}(\vec{u}_{m-1}, \vec{v}_{m-1}) = \frac{\partial u_{m-1}(x,t)}{\partial t} - \alpha \frac{\partial^3 u_{m-1}(x,t)}{\partial x^3} - 6\alpha \sum_{s=0}^{m-1} u_s(x,t) \frac{\partial u_{m-1-s}(x,t)}{\partial x}$$

$$-\beta \sum_{s=0}^{m-1} v_s(x,t) \frac{\partial v_{m-1-s}(x,t)}{\partial x}, \quad (16)$$

$$R_{2m}(\vec{u}_{m-1}, \vec{v}_{m-1}) = \frac{\partial v_{m-1}(x,t)}{\partial t} + \frac{\partial^3 v_{m-1}(x,t)}{\partial x^3} + 3 \sum_{s=0}^{m-1} u_s(x,t) \frac{\partial v_{m-1-s}(x,t)}{\partial x}. \quad (17)$$

and

$$\chi_m = \begin{cases} 0, & m \le 1 \\ 1, & m > 1. \end{cases}$$

Now, the term of the m^{th}-order displacement Equations (14) and (15) for $m \geq 1$ and $H(x,t) = 1$ becomes:

$$u_m(x,t) = \chi_m u_{m-1}(x,t) + hL^{-1}[R_{1m}(\vec{u}_{m-1}(x,t), \vec{v}_{m-1}(x,t))], \quad (18)$$

$$v_m(x,t) = \chi_m v_{m-1}(x,t) + hL^{-1}[R_{2m}(\vec{u}_{m-1}(x,t), \vec{v}_{m-1}(x,t))]. \quad (19)$$

Put $m = 1$; from the Equations (18) and (19), becomes:

$$u_1(x,t) = hL^{-1}\left[R_{11}\left(\vec{u}_0(x,t), \vec{v}_0(x,t)\right)\right], \quad (20)$$

$$v_1(x,t) = hL^{-1}\left[R_{21}\left(\vec{u}_0(x,t), \vec{v}_0(x,t)\right)\right], \quad (21)$$

and equations (16) and (17), becomes:

$$R_{11}(\vec{u}_0, \vec{v}_0) = \frac{\partial u_0(x,t)}{\partial t} - \alpha \frac{\partial^3 u_0(x,t)}{\partial x^3} - 6\alpha u_0(x,t) \frac{\partial u_0(x,t)}{\partial x} - \beta v_0(x,t) \frac{\partial v_0(x,t)}{\partial x}, \quad (22)$$

$$R_{21}(\vec{u}_0, \vec{v}_0) = \frac{\partial v_0(x,t)}{\partial t} + \frac{\partial^3 v_0(x,t)}{\partial x^3} + 3u_0(x,t) \frac{\partial v_0(x,t)}{\partial x}. \quad (23)$$

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Substituting equations (22)-(23) and the initial approximation $u_0(x,t)$ and $v_0(x,t)$ into equations (20)-(21) respectively, to obtain a first approximation $u_1(x,t)$ and $v_1(x,t)$ as follows:

$$u_1(x,t) = h\int_0^t \left(-\alpha \frac{\partial^3 u_0(x,\tau)}{\partial x^3} - 6\alpha u_0(x,\tau)\frac{\partial u_0(x,\tau)}{\partial x} - \beta v_0(x,\tau)\frac{\partial v_0(x,\tau)}{\partial x}\right) d\tau,$$

$$v_1(x,t) = h\int_0^t \left(\frac{\partial^3 v_0(x,\tau)}{\partial x^3} + 3u_0(x,\tau)\frac{\partial v_0(x,\tau)}{\partial x}\right) d\tau,$$

$$u_1(x,t) = h\left(-\alpha \frac{\partial^3 u_0}{\partial x^3} - 6\alpha u_0 \frac{\partial u_0}{\partial x} - \beta v_0 \frac{\partial v_0}{\partial x}\right) t,$$

$$v_1(x,t) = h\left(\frac{\partial^3 v_0}{\partial x^3} + 3u_0 \frac{\partial v_0}{\partial x}\right) t.$$

In the following parts, we consider the initial approximations and determine other components of the solution series for the coupled KdV system.

NUMERICAL RESULTS

This section contains the following information, apply HAM to solve the nonlinear coupled KdV system, and present numerical results to verify the efficacy of this method, we take the following example:

Example:

Consider the following nonlinear coupled KdV system:

$$u_t - \alpha u_{xxx} - 6\alpha u u_x - \beta v v_x = 0,$$

$$v_t + v_{xxx} + 3u v_x = 0,$$

with accurate solutions are provided by:

$$u(x,t) = \frac{1}{3}(-\lambda^2 - 2\mu - \omega) + \frac{4\mu\lambda}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\,(x + x_0 + \omega t)\right) + \lambda}$$

$$- \frac{8\mu^2}{\left(\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\,(x + x_0 + \omega\, t)\right) + \lambda\right)^2},$$

and

$$v(x,t) = \frac{-\lambda\sqrt{-\alpha\lambda^2 + 4\alpha\mu - 2\alpha\omega - \omega}}{\sqrt{2\beta}} + \frac{\mu\sqrt{8(-\alpha\lambda^2 + 4\alpha\mu - 2\alpha\omega - \omega)}}{\sqrt{\beta}\left(\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\,(x + x_0 + \omega\, t)\right)\right)},$$

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where $t = 0$, we get the initial conditions:

$$u_0(x,t) = \frac{1}{3}(-\lambda^2 - 2\mu - \omega) + \frac{4\mu\lambda}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\,(x + x_0)\right) + \lambda}$$
$$- \frac{8\mu^2}{\left(\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\,(x + x_0)\right) + \lambda\right)^2},$$

and

$$v_0(x,t) = \frac{-\lambda\sqrt{-\alpha\lambda^2 + 4\alpha\mu - 2\alpha\omega - \omega}}{\sqrt{2\beta}} + \frac{\mu\sqrt{8(-\alpha\lambda^2 + 4\alpha\mu - 2\alpha\omega - \omega)}}{\sqrt{\beta}\left(\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\,(x + x_0)\right)\right)},$$

where λ and μ are arbitrary constants, α and β are nonzero parameter.

We start from $u_0(x,t)$ and $v_0(x,t)$. By means of the equations (18) and (19), we can easily obtain the other component of the HAM solution directly in the series form of equations (10) and (11). Thus, we have:

$$u_1(x,t) = -\frac{1}{3}\left(\mu(-\lambda^2 + 4\mu)\left(-1 + \tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\,(x + x_0)\right)^2\right)\right)$$
$$\times(-6\alpha\lambda^6\beta - 144\,\alpha^2\lambda^4\mu + 288\,\alpha^2\mu^2\lambda^2 - 96\,\alpha\beta\lambda^2\mu^2 + 48\,\alpha\lambda^4\beta\mu$$
$$+96\,\alpha\mu\beta\omega\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\,(x + x_0)\right)\lambda$$
$$+120\,\alpha\beta\lambda^3\mu\sqrt{\lambda^2 - 4\mu}\,\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\,(x + x_0)\right)$$
$$+48\,\alpha\beta\mu\,\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\,(x + x_0)\right)^3\sqrt{\lambda^2 - 4\mu}$$
$$-192\,\alpha\beta\lambda\mu^2\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\,(x + x_0)\right)$$
$$-96\,\alpha\beta\lambda\mu^2\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\,(x + x_0)\right)^3\sqrt{\lambda^2 - 4\mu}$$

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$$-36\alpha\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)\lambda^3\beta\omega$$

$$+18\beta^{3/2}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\lambda^2\mu\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)$$

$$-3\beta^{\frac{3}{2}}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\,\omega\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)\lambda^2$$

$$-12\alpha\lambda^4\beta\omega+10\beta^{\frac{3}{2}}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\lambda^3\mu$$

$$-\beta^{\frac{3}{2}}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\,\omega\lambda^3-24\,\beta^{\frac{3}{2}}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\,\mu^2\lambda$$

$$-18\,\alpha\beta\lambda^6\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)+384\,\alpha\beta\mu^3\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2$$

$$-144\,\alpha^2\lambda^2\mu\omega-72\alpha\lambda^2\mu\omega-504\,\alpha^2\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda^4\mu$$

$$+1440\,\alpha^2\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda^2\mu^2+108\alpha^2\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda^4\omega$$

$$+54\alpha\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda^4\omega+54\alpha^2\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)\lambda^5$$

$$+576\,\alpha^2\mu^2\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\omega+288\,\alpha\mu^2\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\omega$$

$$+18\,\alpha^2\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3\lambda^5$$

$$-3\beta^{3/2}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda^5$$

$$-\beta^{\frac{3}{2}}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\lambda^5+36\alpha^2\lambda^4\omega+18\alpha\lambda^4\omega$$

$$+54\alpha^2\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda^6-1152\alpha^2\mu^3\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2$$

$$+18\alpha^2\lambda^6+48\alpha\mu\beta\omega\lambda^2-18\alpha\beta\lambda^5\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)$$

$$-6\alpha\beta\lambda^5\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3$$

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$$+168\,\alpha\beta\lambda^4\mu\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2$$

$$-480\,\alpha\beta\lambda^2\mu^2\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2$$

$$-2\beta^{\frac{3}{2}}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\,\lambda^4\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)$$

$$-24\,\beta^{\frac{3}{2}}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\,\mu\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)$$

$$-12\alpha\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3\lambda^3\beta\omega$$

$$-144\,\alpha^2\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3\lambda\omega\mu$$

$$-72\alpha\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3\lambda\omega\mu$$

$$+192\,\alpha\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda^2\beta\omega\mu$$

$$-288\,\alpha\mu\sqrt{\lambda^2-4\mu}\tanh\left(\tfrac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)$$

$$+18\,\beta^{\frac{3}{2}}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\mu\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda^3$$

$$-24\beta^{\frac{3}{2}}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\mu\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda$$

$$-3\beta^{\frac{3}{2}}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\,\omega\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda^3$$

$$+8\beta^{\frac{3}{2}}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\,\mu^2\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3$$

$$-\beta^{\frac{3}{2}}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\,\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3\lambda^4$$

$$-144\,\alpha^2\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3\lambda^3\mu$$

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$$+288\alpha^2\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3\lambda\mu^2$$

$$+36\,\alpha^2\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3\lambda^3\omega$$

$$+18\,\alpha\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3\lambda^3\omega$$

$$-36\alpha\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda^4\beta\omega$$

$$-192\,\alpha\mu^2\beta\omega\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2$$

$$+576\,\alpha^2\mu^2\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)\lambda$$

$$-576\,\alpha^2\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda^2\omega\mu$$

$$-288\,\alpha\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda^2\omega\mu$$

$$-360\alpha^2\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)\lambda^3\mu$$

$$+108\,\alpha^2\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)\lambda^3\omega$$

$$+54\,\alpha\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)\lambda^3\omega$$

$$+48\,\alpha\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3\lambda\beta\omega\mu$$

$$-144\alpha\mu\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)\lambda\omega$$

$$+2\beta^{3/2}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\,\mu\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3\lambda^2\mu$$

$$-\beta^{3/2}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3\lambda^2\omega$$

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$$+4\beta^{3/2}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^3\omega\mu$$

$$+12\beta^{3/2}\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\,\tanh\left(\tfrac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\lambda\mu\Big)\,t\,/$$

$$\left(\left(\sqrt{\lambda^2-4\mu}\,\tanh\left(\tfrac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)+\lambda\right)^5\beta\right).$$

$$v_1(x,t)=\Bigg(\bigg(12\lambda^2\cosh\left(\tfrac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2\mu+8\sqrt{\lambda^2-4\mu}\sinh\left(\tfrac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)\lambda\cosh\left(\tfrac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)\mu-$$

$$2\sqrt{\lambda^2-4\mu}\,\sinh\left(\tfrac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)\lambda^3\cosh\left(\tfrac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)-$$

$$2\lambda^4\cosh\left(\tfrac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2+\lambda^4-16\mu^2\cosh\left(\tfrac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)^2+16\mu^2-$$

$$8\lambda^2\mu\,\omega\bigg)\omega\,\mu\sqrt{-2\alpha\lambda^2+8\alpha\mu-4\alpha\omega-2\omega}\;t\Bigg)\Big/\left(\sqrt{\beta}\left(\sqrt{\lambda^2-4\mu}\sinh\left(\tfrac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)+\lambda\cosh\left(\tfrac{1}{2}\sqrt{\lambda^2-4\mu}\,(x+x_0)\right)\right)^4\right),$$

Similarly, the remaining components of the iteration formulae (18) and (19), the Maple package can be used to collect this data.

Thus $u(x,t)$ and $v(x,t)$ can be expressed as follows

$$u(x,t)=u_0(x,t)+u_1(x,t)+u_2(x,t)+\cdots,$$
$$v(x,t)=v_0(x,t)+v_1(x,t)+v_2(x,t)+\cdots.$$

It is clear that the recurrence relation (18) and (19), can be find all the information of the components $u(x,t)$ and $v(x,t)$, to obtained the analytical solution.

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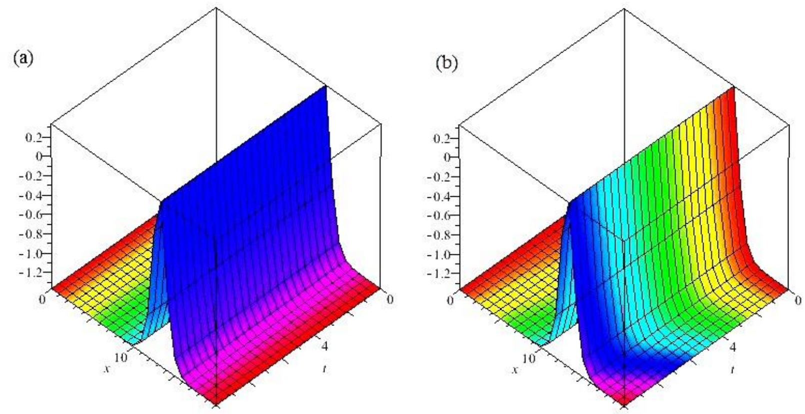

Figure 1. Plots of results of Example above, when $0 \leq x \leq 20, 0 \leq t \leq 10$, $\alpha = -2.5$, $\beta = 2.5, x_0 = -15, \omega = 0.5, \mu = 0.1$ and $\lambda = 2$.

(a) Exact solution of $u(x,t)$,

(b) Approximate solution of second-order of $u(x,t)$ by HAM with $h = -1$.

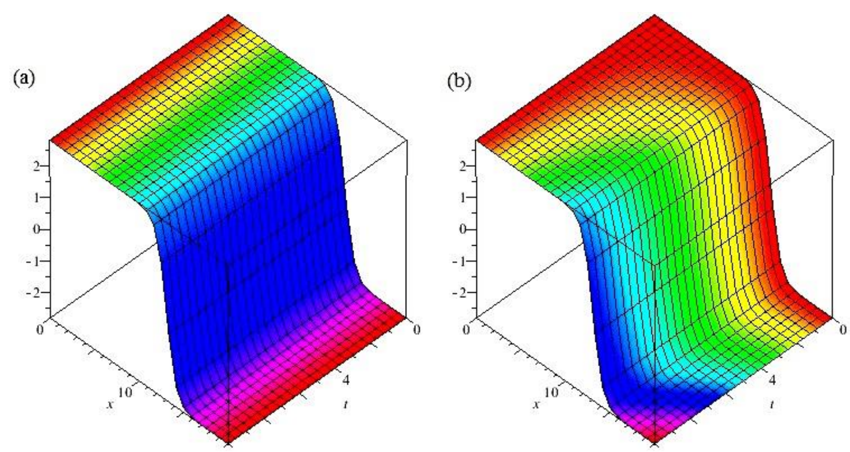

Figure 2. Plots of results of Example above, when $0 \leq x \leq 20, 0 \leq t \leq 10$, $\alpha = -2.5$, $\beta = 2.5, x_0 = -15, \omega = 0.5, \mu = 0.1,$ and $\lambda = 2$.

(a) Exact solution of $v(x,t)$,

(b) Approximate solution of second-order of $v(x,t)$ by HAM with $h = -1$.

Conclusion:

In this work, we've completed our aim reformulate homotopy analysis method to acquire approximations of the coupled KdV system's solutions. More precisely, Figures 1 and 2 show that near closeness between the accurate and numerical solutions obtained. The results demonstrate accurate and effectual of the homotopy method for the resolution of the coupled KdV system.

Conflict of interests.

There are non-conflicts of interest.

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الخلاصة

يتم تقديم طريقة (HAM) تحليلية لحل نظام (KdV)غير الخطي المقترن. يحتوي (HAM)على معامل مساعد h ، والذي يعطي طريقة مفيدة للتحكم في لغة الدمج لترتيبات الترتيب .تظهر المقارنات العددية التي تم عرضها في جداول بشكل موحد بيانيا في اشكال و توضح و تؤكد الدقة الطويلة وتحدد إنتاجية و استقامه الاستراتيجية.

الكلمات الدالة: طريقة التحليل هوموتوبي ،النظام (KdV) ثنائي ، الحل التحليلي.

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