

(n-1) – Reconstruction of Tournaments M_n

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Abstract

In this paper, we show that the kind of tournament M_n , which are indecomposable, then this type of tournament are (n-1) RECONSTRUCTION.

Key words:

Reconstruction, Tournament, Indecomposable, Homorphic, Diamond, Isomorphic.

الخلاصة

في هذا البحث وجدنا ان العلاقة الدورية الدائرية من نوع M_n والتي تتصف بكونها غير قابلة للفصل تكون (1-n) قابلة لإعادة البناء.

الكلمات المفتاحية:

إعادة البناء، العلاقة الدورية الدائرية، العلاقة الغير قابلة للتجزئة، تشاكل، علاقة الماسة.

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INTRODUCTION

Reconstruction questions arise when studying interaction between the isomorphic type of a structure and the isomorphic type of its substructures. In this paper we are interested binary relations. We present most of the known results on different kinds of reconstruction: among them we have the Fraïssé-reconstruction, the Ulam-construction, the max-reconstruction and the set-reconstruction.

For any indecomposable tournament M_n of finite cardinal n we show that this kind of tournaments are $(n-1)$ RECONSTRUCTION, let $1, 2, 3, \dots, n$ the vertices of M_n , then $M_n - \{1\}$ and $M_n - \{n\}$ are isomorphic to M_{n-1} which is also indecomposable. For $x \neq 1, n$, $M_n - \{x\}$ is decomposable and equal to the sum of M_p and M_q with $p < n-1$, $q < n-1$. Let M_n tournament $(n-1)$ homomorphic to M_n [4].

M_n is indecomposable then M is isomorphic to M_n because the M_n are characterized by their bounds of cardinal 5: A_1, A_2, A_3, A_4, A_5 and M admit the same bounds for $n \geq 5$ [3].

Definitions:

- 1 – A tournament M_n consists of finite set v of vertices with a prescribed collection A of ordered pairs of distinct vertices, called the set of arcs of T , which satisfies: for $x, y \in v$, with $x \neq y$, $(x, y) \in A$ if and only if $(y, x) \notin A$. Such a tournament M_n is denoted by (V, A) [1].
- 2 – the tournaments M_n are defined on the base $E = \{1, 2, 3, \dots, n\}$ such that $M_n(i, j) = +$ if and only if $i < j-1$ or $i=j+1$. We note that M_4 is a 4-cycle and M_3 is a 3-cycle. M_n have the following **Properties** [1].
 - A) M_n is isomorphic to its converse [1].
 - B) M_n is strong i.e. that for all x, y there exist a walk from x to y [2].
- 3 – indecomposable tournament or strong tournament is a tournament that we can not partition its base E in interval E_i , such that one of them is of cardinal ≥ 2 [1].
- 4 – A tournament T is $(n-1)$ -homomorphic to the tournament T (T and T have the same base E), where $\forall x \in E$ then $T/E-x$ is isomorphic to $T/E-x$ [4].
- 5 – A tournament T is said to be $(n-1)$ -reconstructible where each tournament T $(n-1)$ -homomorphic to T is isomorphic to T [5].

Let the base $A = \{0, 1, 2, 3, 4\}$ by dilating a 3-cycle we get a diamond A_5 is a positive diamond if $A_5 / \{0, 1, 2, 3\}$ IS A NEGATIVE DIAMOND of vertex and $A_5(2, 4) = +$

We get the diamonds by deleting the 3-cycle by one and only one for the two points from the chain . to two element's 1, 2 (positive if the vertex is 1 and negative if the vertex 2).

(n-1) RECONSTRUCTION of M_n

Theorem :

Let $1, 2, 3, \dots, n$ be the vertices of M_n then $M_n - \{1\}$ and $M - \{n\}$ are isomorphic to M_{n-1} which is indecomposable. $\forall x \neq 1, n$ $M_n - \{x\}$ is decomposable and equal to the sum of M_p and M_q with $p < n-1, q < n-1$

Proof: let M Be a tournament $(n-1)$ -homomorphic to M_n

- if M Is indecomposable then M IS ISOMORPHIC TO M_n Because the M_n are characterizes by their bounds of cardinal 5, A_1, A_2, A_3, A_4, A_5 and M - admits the same bounds for $n \geq 5$

- if M is decomposable = let A, B two intervals of \succ . THEN $M - \{1\}$ must be isomorphic to M_{n-1} which is indecomposable, and if 1 belong to A , then $A - \{1\}$ must be vide, if none, we have again two intervals and $M - \{1\}$ become decomposable (contradiction).

Then $A - \{1\} = \varnothing$ and $A = \{1\}$

By the same method with n

If n belong B (for example) then $B = \{n\}$ either we have a third interval c , in this case we delete one element x from c , we have by $(n-1)$ -hypomorphic : $M - \{x\}$ Isomorphic to $M_{n-1} - \{x\}$ which is admit only two intervals then $M - \{x\}$ Admet Also two intervals i. e A, B then $c = \{x\}$ and cardinal of $M = 3$ (contradiction) or there is not a third interval and $M = A \cup B$ (impossible) then M is indecomposable and M - isomorphic To M_n

Note: M_n is a finite tournament (its base is finite, each element of M_n be delete by a set c_i to obtain the tournament M_n (the c_i are singletons in the case or the dilatation is trivial)

Characterization the classes R/ C_i

Proposition1: let R be a tournament which embedded a 3-cycle, and not embedded to 4-cycle , then R obtained by take the chain by delete some points by 3-cycle.

Proof: let C be a 3-cycle and $x \in c$ we can see that $R/ C \cup \{ X \}$ is a diamond

Let $a \rightarrow b \rightarrow c \rightarrow a$ be a 3-cycle C suppose $R(x, a) = +$ then $R(x, b) = +$ if none $R/ \{ x, b, c \}$ is a 3-cycles, the we can partition $E-C$ in two sets $D^+ = \{ X = X \rightarrow C \}$ and $D^- = \{ X = C \rightarrow X \}$ Let $a \in D^+$ and $b \in D^-$ and $u, v \in C$, the we have $R(a, b) = +$ if none $R/ \{ a, b, u \}$ and $R/ \{ a, b, u \}$ must be 3-cycles and $R/ \{ a, b, u, v \}$ must then 4-cycle (contradiction)

Corollary:

first we define the form of the indecomposable class R/ C_i which neither 4-cycle nor chain and nor 3-cycle we can verify that

- 1) $| C_i | \neq 3$, because if none then R/ C_i must indecomposable which take the form of 3-cycle.
- 2) $| C_i | \neq 4$ if none R/ C_i must be a diamond, decomposable which is not cycle and its vertex.
- 3) if $| C_i | > 5$, then R/ C_i is a M_k either it is decomposable

- case 1: $| C_i | = 5$

-Either $R/C_i > 4$ -cycle , then R/C_i not a chain then there exist in R/C_i a 3-cycle a, b, c by proposition 1 , R/C_i is decomposable (contradiction)

-or R/C_i embed 4-cycle by theorem, either $R/C_i \cong M_5$ (the symbol \cong means isomorphic) or R/C_i is decomposable (impossible)

Collar ely= if $| C_i | = 5$ then $R/C_i = M_5$

Case 2 = $| C_i | = k > 5$

-either R/C_i embedded 4-cycle will be indecomposable and of cardinal > 4 , then R/C_i is a M_k (by theorem)

-or R/C_i not embed 4-cycle and $| C_i | \geq 3$ by (proposition) the solution for R/C_i will be decomposable (impossible)

Proposition 2 : if R is a solution decomposable i.e. $(R \succ A_1, A_2, A_3, A_4, A_5)$ THEN R take the form of a finite chain C .

Proof: R is decomposable, there exist a relation c not singleton and different from R , such that $R = D(c)$ (i.e. R is delete from c) suppose that c not a chain i.e. there exist a, b, c from c such that $c/\{a, b, c\}$ is a 3-cycle we delete a, b, c respectively in A, B, C suppose that $|A| \geq 3$, and let $X \in B$, $y \in c$ let $z_1, z_2, z_3 \in A$ then $R/\{x, y\} z_1, z_2, z_3 \in N_{A_3}$ or A_1 , and $R/\{z_1, z_2, z_3\}$ is a chain or a 3-cycle, if cardinal of B or C is ≥ 3 then $\text{card } x < 3$, for $x = A, B, C$

-if $\text{card } A = \text{card } B = 2$, with $x \in A$ we have $R/A \cup B \cup \{X\} \in N_{A_2}$, then the only possibility are $|A| = 2$, $|B| = 1$, $|C| = 1$ or $|A| = |B| = |C| = 1$

Conflict of interests.

There are non-conflicts of interest.

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